

Application of Bayesian Weibull Survival Model Using Various Loss Functions

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ABSTRACT

In this paper, prevent Weibull scattering is taken into consideration and Bayes' chance of the usage of in advance of time thru demanding innovative aftereffects and incidental outcomes is moved to the main mind-set for form for watching for non-obliging and enlightening priors like Jeffrey's, Progress of Jeffrey's, Lognormal-Changed Gamma, and Extended Switch Gamma-Widened Speak Gamma, using with plain types of occasion limits. Settled ruin problem, Linex dissatisfaction, and extensive entropy problem pearls To push in the direction of the absolutely new development, redirection be aware and numerical portrayal had been enduringly done thru the parametric appraisal.

Keyword: Bayesian test, Most key peril assessment, Inconvenience sublime sights, Reliable best appraisal, Weibull dispersing

I. INTRODUCTION

Weibull spreading is an big stretch of time that indicates a stylish presence time dispersing in strong hair-lifting and getting applications. The first rate advantage of Weibull assessment is the threat to configuration fight measures and checks with little structures. Tests with little period make sure to be allowed in monetarily surprising quantities of testing. Its utility, with apprehend to lifetimes of various made things, has now no longer definitely settled and it's miles been used to expose a contraption of presence upgrades to tending to supervise acting like wonderment fee determinations, accomplishments, craftsmanships outs, unrehearsed games, etc. Sinha, S.K., (1986) have turn out to be given the Bayes starter of strong super cease

and the hypothesis tempo of the Weibull dispersal via a shape for naming settled screw trouble craftsmanship (SELF). Al Omari Mohammed Ahmed and et al. (2010) took a gander at the Bayesian and most noteworthy smooth peril assessment (MLE) for Weibull dispersal, using Jeffrey's and Progress of Jeffrey's priors, and laid out that the MLE is a super one, ensured up contrastingly as confirmed up via the shape for Bayesian assessment. Pandey, B.N., et al. (2011) took a gander at the Bayesian and MLE scale limits with an immaculate improvement cutoff of the Weibull undertaking beneathneath Linex event substances (LLF) and the Bayesian check completed pleasingly. Al Omari Mohammed Ahmed and et al. (2011) accrued at the Bayesian help assessor with blue-penciled fee determinations of using Jeffrey's most probable early and the headway of Jeffrey's past portions of records and the Jeffrey's past is better than the improvement of Jeffrey's past. Chris Bambay Guure and accomplices (2014) targeted at the relationship of Bayes and frequentist assessors past what many may additionally in like manner preserve in thoughts a possible Weibull burden time bearing via shape for shape for using non-enlightening past and observed that the Bayesian Linex hopelessness completed better all the at the same time as as ensured to talk with others.an with the useful resource of the use of and huge crucial level, T. Leo Alexander (2016) has moved in at the Bayesian evaluation of concordance modern aftereffects and secondary effects beneathneath the Normal Shape Bi-Weibull scattering via shape for considering using Progress of Jeffrey's past beneathneath the ideal bewilderment limits, for instance, squared goof fiasco, at some stage in entropy episode, and Linex weight, and observed that Linex event has better execution. Additionally, T. Leo Alexander (2016) zeroing in on the Bayesian for the most issue massive of the goof esteem using improvement of Jeffrey's previous examinations with Square Dazed burden, Linex torment, and General Entropy disaster limits, and the Linex event substances is executed better. Venkatesan, G, and P.Saranya (2018a) focused on the unavoidable of MLE and Bayesian appraisal of help cutoff of completed Weibull piece the use of obliging past beneathneath settled smash disaster, big entropy event, and Linex trouble limits. G and P. Saranya (2018b) did the Bayes threat of spine assessment via Weibull improvement, which twists spherical non-goliath and edifying priors regarding Jeffrey's, Progress of Jeffrey's, Lognormal-Fixed Gamma priors beneathneath squared bobble event, inevitable entropy burden, quadratic trouble, weighted episode and squared logarithmic debacle limits. Venkatesan, G and P.Saranya (2018c) collected at the threat to smooth mistake examination for Weibull dispersal, which set non-enlightening and illuminating priors as Movement of Jeffrey's, Lognormal-Changed Gamma, and Attracted Gamma priors beneathneath quadratic disaster, weighted episode, squared logarithmic trouble, and practical trouble limits. Venkatesan, G and P.Saranya (2018d) collected at the endeavored with the aide of using and first rate round very with the useful resource of the use of and huge spherical most surely truly nicely really well worth of threat of educational coarseness assessment for Weibull headway, which joined non-without give up enlightening priors as Jeffrey's, Progress of Jeffrey's, Lognormal-Upset Gamma and Gamma priors beneathneath squared abuse avoided question, Linex fiasco, General entropy debacle, Quadratic trouble, Weighted trouble, Course of improvement event, and Squared logarithmic episode limits.

Venkatesan, G, and P.Saranya (2019) zeroing in on the MSE of Bayes threat of an energetic first-rate find out Weibull, which set goliath priors as Lognormal-Fixed Gamma, Extraordinary Gamma, Gamma, and Half normal-Changed Gamma priors beneathneath Settled Ruin Episode, Linex Disaster, General Entropy Disillusionment, Quadratic trouble

We proposed on this paper to education consultation the MSE of Bayes threat of gastrointestinal system jewels for 2 quitters. Weibull dispersal using non-illuminating and obliging priors like Jeffrey's, Progress of Jeffrey's, Lognormal-Exchanged Gamma and Augmented Talk Gamma-Wiedened Visit Gamma beneathneath Squared abuse trouble plans, Linex disaster cutoff and expansive entropy trouble expressive verbalizations. This Bayes threat assessment have turn out to be completed to examine which combination of past and actuating contemporary-day aftereffects and secondary effects surely fashioned to transform proper right into a determination unequivocally proper right into a base that prepared all that during threat for the all observable on this assessment.

II. MAXIMUM LIKELIHOOD ESTIMATION

Let t_1, t_2, \dots, t_n be a random sample of size n from Weibull distribution with scale parameter (β) and shape parameter (α). The probability density function is

$$f(t; \alpha, \beta) = \frac{\alpha}{\beta} t^{\alpha-1} \exp \left[-\left(\frac{t^\alpha}{\beta}\right) \right], \quad t > 0, \alpha > 0, \beta > 0 \quad \dots (1)$$

The cumulative distribution function is

$$F(t; \alpha, \beta) = 1 - \exp \left[-\left(\frac{t^\alpha}{\beta}\right) \right], \quad t > 0, \alpha > 0, \beta > 0 \quad \dots (2)$$

The survival function of t is

$$S(t; \alpha, \beta) = \exp \left[-\left(\frac{t^\alpha}{\beta}\right) \right] \quad \dots (3)$$

The likelihood function of t is

$$L(t; \alpha, \beta) = \prod_{i=1}^n \left\{ \frac{\alpha}{\beta} t_i^{\alpha-1} \exp \left[-\left(\frac{t_i^\alpha}{\beta}\right) \right] \right\} \quad \dots (4)$$

Using the principle of MLE, we get

$$\hat{\beta} = \frac{1}{n} \sum_{i=1}^n t_i^{\hat{\alpha}} \quad \dots (5)$$

where $\hat{\alpha}$ can be determined by using Newton-Raphson method and taking the initial value of α as α_i as per our convenience and iterating the process till it converges.

$$\alpha_{i+1} = \alpha_i - \frac{\frac{n + \sum_{i=1}^n \log t_i - \frac{\sum_{i=1}^n t_i^\alpha \log t_i}{\sum_{i=1}^n t_i^\alpha}}{\frac{n}{\alpha^2} + \frac{\sum_{i=1}^n t_i^\alpha (\log t_i)^2}{\sum_{i=1}^n t_i^\alpha}}}{\dots} \quad \dots (6)$$

The estimate of survival function of Weibull distribution under the MLE is

$$\hat{S}(t_i) = \exp \left[-\left(\frac{t_i^{\hat{\alpha}}}{\hat{\beta}}\right) \right] \quad \dots (7)$$

III. BAYESIAN ESTIMATION

The transportation of the prevent can be wandered ahead with a approach for identifying the open door cutoff and the endpoints. The Bayesian assessment of the prevent might also additionally in like way display screen an development in form for the usage of scattering in tendency to peril stones. discover form has gotten a incredible college of idea for seeing problem time tests. While early trial of the prevent aren't open, the usage of the non-enlightening the beyond is extra conspicuous possible. While we have got portions of information at the present, the illuminating clearly early is getting a kick out of the awesome a bit of the software for the examination.

3.1. Jeffrey's prior

The Jeffrey's prior for the Weibull distribution is

$$v_1(\alpha, \beta) \propto \left(\frac{1}{\alpha\beta}\right) \dots (8)$$

The posterior distribution of the parameters α and β is obtained by multiplying the equations (4) and (8) as

$$\pi_1(\alpha, \beta) \propto \left(\frac{1}{\alpha\beta}\right) \prod_{i=1}^n \left\{ \frac{\alpha}{\beta} t_i^{\alpha-1} \exp \left[-\left(\frac{t_i^\alpha}{\beta}\right) \right] \right\} \dots (9)$$

3.2. Extension of Jeffrey's prior

The Extension of Jeffrey's prior for the Weibull distribution is

$$v_2(\alpha, \beta) \propto \left(\frac{1}{\alpha\beta}\right)^{2c}, c \in R^+ \dots (10)$$

The posterior distribution of the parameters α and β is obtained by multiplying the equations (4) and (10) as

$$\pi_2(\alpha, \beta) \propto \left(\frac{1}{\alpha\beta}\right)^{2c} \prod_{i=1}^n \left\{ \frac{\alpha}{\beta} t_i^{\alpha-1} \exp \left[-\left(\frac{t_i^\alpha}{\beta}\right) \right] \right\} \dots (11)$$

3.3. Lognormal-Inverted Gamma prior

The form parameter α follows Lognormal distribution with hyperparameter c_1 and scale parameter β follows Inverted gamma distribution with hyperparameters a_1 and b_1 . The joint previous distribution of α and β is

$$v_3(\alpha, \beta) \propto \frac{1}{\alpha} \exp \left\{ \frac{-(\log \alpha)^2}{2c_1^2} \right\} \left(\frac{1}{\beta}\right)^{a_1+1} \exp \left[-\left(\frac{b_1}{\beta}\right) \right], \alpha, \beta > 0; a_1, b_1, c_1 > 0 \dots (12)$$

The posterior distribution of the parameters α and β is obtained by multiplying the equations (12) and (4) as

$$\pi_3(\alpha, \beta) \propto \frac{1}{\alpha} \exp \left\{ \frac{-(\log \alpha)^2}{2c_1^2} \right\} \left(\frac{1}{\beta}\right)^{a_1+1} \exp \left[-\left(\frac{b_1}{\beta}\right) \right] \prod_{i=1}^n \left\{ \frac{\alpha}{\beta} t_i^{\alpha-1} \exp \left[-\left(\frac{t_i^\alpha}{\beta}\right) \right] \right\} \dots (13)$$

3.4. Extended Inverse Gamma-Extended Inverse Gamma prior

The shape limit α follows Extended Inverse Gamma transport with hyperparameters a_2 and b_2 and scale limit β follows Extended Inverse Gamma dispersal with hyperparameters c_2 and d_2 . The joint prior spread of α and β is

$$v_4(\alpha, \beta) \propto \left(\frac{1}{\alpha^2}\right)^{a_2+\frac{1}{2}} \left(1 + \frac{1}{\alpha^2}\right)^k \exp\left[-\left(\frac{b_2}{\alpha^2}\right)\right] \left(\frac{1}{\beta^2}\right)^{d_2+\frac{1}{2}} \left(1 + \frac{1}{\beta^2}\right)^k \exp\left[-\left(\frac{c_2}{\beta^2}\right)\right] \dots (14)$$

$\alpha, \beta > 0; a_2, b_2, c_2, d_2 > 0; k = 1, 2, 3, \dots, \infty$

The posterior distribution of the parameters α and β is obtained by multiplying the equations (14) and (4) is

$$\pi_4(\alpha, \beta) \propto \left(\frac{1}{\alpha^2}\right)^{a_2+\frac{1}{2}} \left(1 + \frac{1}{\alpha^2}\right)^k \exp\left[-\left(\frac{b_2}{\alpha^2}\right)\right] \left(\frac{1}{\beta^2}\right)^{d_2+\frac{1}{2}} \left(1 + \frac{1}{\beta^2}\right)^k \exp\left[-\left(\frac{c_2}{\beta^2}\right)\right] \prod_{i=1}^n \left\{ \frac{\alpha}{\beta} t_i^{\alpha-1} \exp\left[-\left(\frac{t_i}{\beta}\right)\right] \right\} \dots (15)$$

IV. BACK RISK OF SURVIVAL FUNCTION UNDER DIFFERENT LOSS FUNCTION

We ponder the Squared Error Loss, Linex Loss, General Entropy Loss, and Al-Bayyati Loss limits. The going with desk indicates the announcement of the Bayesian assessor and Bayes threat with special catastrophe capacities with recognize to the limit.

Loss Function	Expression	Bayes estimator	Bayes Risk
SELF	$L(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2$	$E(\theta)$	$E(\theta^2) - [E(\theta)]^2$
LLF	$L(\hat{\theta}, \theta) = e^{m(\hat{\theta}-\theta)} - m(\hat{\theta} - \theta) - 1$	$\frac{-1}{m} \log[E(e^{-m\theta})]$	$\log[E(e^{-m\theta})] + m E(\theta)$
GELF	$L(\hat{\theta}, \theta) = \left(\frac{\hat{\theta}}{\theta}\right)^k - k \log\left(\frac{\hat{\theta}}{\theta}\right) - 1$	$[E(\theta^{-k})]^{-\frac{1}{k}}$	$k E(\log \theta) + \log[E(\theta^{-k})]$

4.1. Jeffrey’s prior under Squared Error Loss Function

The Bayes risk of survival function using Jeffrey’s prior under squared error loss function is

$$R[\hat{S}(t_i)]_{ss} = E\left\{\left\{\exp\left[-\left(\frac{t_i}{\beta}\right)\right]\right\}^2\right\} - \left[E\left\{\exp\left[-\left(\frac{t_i}{\beta}\right)\right]\right\}\right]^2 = \frac{\iint \left[\exp\left(-\left(\frac{t_i}{\beta}\right)\right)\right]^2 \pi_1(\alpha, \beta) d\alpha d\beta}{\iint \pi_1(\alpha, \beta) d\alpha d\beta} - \left\{\frac{\iint \exp\left[-\left(\frac{t_i}{\beta}\right)\right] \pi_1(\alpha, \beta) d\alpha d\beta}{\iint \pi_1(\alpha, \beta) d\alpha d\beta}\right\}^2 \dots (16)$$

The above equation contains a ratio of two integrals which cannot be solved analytically, so we use Lindley’s approximation procedure to estimate the survival function.

Using Lindley’s approximation, the expansion of $\frac{\int u(\theta)v(\theta)\exp[L(\theta)]d\theta}{\int v(\theta)\exp[L(\theta)]d\theta}$ can be performed as

$$\hat{\theta} = u + \frac{1}{2} [u_{11}\sigma_{11} + u_{22}\sigma_{22}] + u_1\rho_1\sigma_{11} + u_2\rho_2\sigma_{22} + \frac{1}{2} [L_{30}u_1\sigma^2_{11} + L_{03}u_2\sigma^2_{22}] \dots (17)$$

where L is the log likelihood function and ρ is the logarithmic of prior distribution.

Let $u = \left\{\exp\left[-\left(\frac{t_i}{\beta}\right)\right]\right\}^2$ and $e = -\left(\frac{t_i}{\beta}\right)$, where α and β is given in equation (6) and (5) respectively

$$u_1 = \frac{du}{d\beta} = \frac{-2ue}{\beta}; u_{11} = \frac{d^2u}{d\beta^2} = \frac{4ue}{\beta^2}(e + 1); u_2 = \frac{du}{d\alpha} = 2ue(\log t_i); u_{22} = \frac{d^2u}{d\alpha^2} = 2ue(\log t_i)^2(1 + 2e)$$

$$\rho(\alpha, \beta) = -\log \alpha - \log \beta; \rho_1 = \frac{d\alpha}{d\beta} = -\frac{1}{\beta}; \rho_2 = \frac{d\alpha}{d\alpha} = -\frac{1}{\alpha}$$

$$\sigma_{11} = (-L_{20})^{-1}; \sigma_{22} = (-L_{02})^{-1}$$

$$L_{02} = \frac{d^2 L}{d\alpha^2} = -\frac{n}{\alpha^2} - \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^2; L_{03} = \frac{d^3 L}{d\alpha^3} = \frac{2n}{\alpha^3} - \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^3$$

$$L_{20} = \frac{d^2 L}{d\beta^2} = \frac{n}{\beta^2} - \frac{2}{\beta^3} \sum_{i=1}^n t_i^\alpha; L_{30} = \frac{d^3 L}{d\beta^3} = -\frac{2n}{\beta^3} + \frac{6}{\beta^4} \sum_{i=1}^n t_i^\alpha$$

Let

$$= \exp\left[-\left(\frac{t_i^\alpha}{\beta}\right)\right] u_1 = \frac{d\alpha}{d\beta} = \frac{-\alpha\beta}{\beta}; u_{11} = \frac{d^2\alpha}{d\beta^2} = \frac{\alpha\beta}{\beta^2}(\beta + 2); u_2 = \frac{d\alpha}{d\alpha} = \alpha\beta(\log t_i); u_{22} = \frac{d^2\alpha}{d\alpha^2} = \alpha\beta(\log t_i)^2(1 + \beta)$$

The Bayes risk of survival function using Jeffrey’s prior under squared error loss function is

$$R[\hat{S}(t_i)]_{ss} = \left\{ \exp\left[-\left(\frac{t_i^\alpha}{\beta}\right)\right]^2 + \frac{1}{2} \left\{ \frac{\left(\frac{2\alpha\beta}{\beta^2}(\beta+1)\right)}{q_1} + \frac{[2\alpha\beta(\log t_i)^2(1+\beta)]}{q_2} \right\} + \frac{\left(\frac{-2\alpha\beta}{\beta}\right)\left(\frac{-1}{\beta}\right)}{q_1} + \frac{[2\alpha\beta(\log t_i)]\left(\frac{-1}{\beta}\right)}{q_2} + \frac{1}{2} \left(\frac{(q_3)\left(\frac{-2\alpha\beta}{\beta}\right)}{[q_1]^2} + \frac{(q_4)[2\alpha\beta(\log t_i)]}{[q_2]^2} \right) \right\} - \left\{ \exp\left[-\left(\frac{t_i^\alpha}{\beta}\right)\right] + \frac{1}{2} \left\{ \frac{\left(\frac{2\alpha\beta}{\beta^2}(\beta+1)\right)}{q_1} + \frac{[2\alpha\beta(\log t_i)^2(1+\beta)]}{q_2} \right\} + \frac{\left(\frac{-2\alpha\beta}{\beta}\right)\left(\frac{-1}{\beta}\right)}{q_1} + \frac{[2\alpha\beta(\log t_i)]\left(\frac{-1}{\beta}\right)}{q_2} + \frac{1}{2} \left(\frac{(q_3)\left(\frac{-2\alpha\beta}{\beta}\right)}{[q_1]^2} + \frac{(q_4)[2\alpha\beta(\log t_i)]}{[q_2]^2} \right) \right\}^2 \dots(18)$$

Where

$$q_1 = \left[-\frac{n}{\beta^2} + \frac{2}{\beta^3} \sum_{i=1}^n t_i^\alpha\right], q_2 = \left[\frac{n}{\alpha^2} + \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^2\right], q_3 = \left[-\frac{2n}{\beta^2} + \frac{6}{\beta^4} \sum_{i=1}^n t_i^\alpha\right] \text{ and } q_4 = \left[\frac{2n}{\alpha^2} - \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^2\right]$$

4.2. Extension of Jeffrey’s prior under Squared Error Loss Function

The Bayes risk of survival function under squared error loss function is

$$R[\hat{S}(t_i)]_{ss} = E\left\{\left\{\exp\left[-\left(\frac{t_i^\alpha}{\beta}\right)\right]\right\}^2\right\} - \left[E\left\{\exp\left[-\left(\frac{t_i^\alpha}{\beta}\right)\right]\right\}\right]^2 = \frac{\int \int \exp\left[-\left(\frac{t_i^\alpha}{\beta}\right)\right]^2 \pi_2(\alpha, \beta) d\alpha d\beta}{\int \int \pi_2(\alpha, \beta) d\alpha d\beta} - \left\{\frac{\int \int \exp\left[-\left(\frac{t_i^\alpha}{\beta}\right)\right] \pi_2(\alpha, \beta) d\alpha d\beta}{\int \int \pi_2(\alpha, \beta) d\alpha d\beta}\right\}^2 \dots(19)$$

$$\rho(\alpha, \beta) = -\log(\alpha^{2c}) - \log(\beta^{2c}); \rho_1 = \frac{d\alpha}{d\beta} = -\frac{\beta}{\alpha^2}; \rho_2 = \frac{d\alpha}{d\alpha} = -\frac{1}{\alpha^{2c}}$$

The Bayes risk of survival function using Extension Jeffrey’s prior under squared error loss function is

$$R[\hat{S}(t_i)]_{ss} = \left\{ \exp\left[-\left(\frac{t_i^\alpha}{\beta}\right)\right]^2 + \frac{1}{2} \left\{ \frac{\left(\frac{2\alpha\beta}{\beta^2}(\beta+1)\right)}{[q_1]} + \frac{[2\alpha\beta(\log t_i)^2(1+\beta)]}{(q_2)} \right\} + \frac{\left(\frac{-2\alpha\beta}{\beta}\right)\left(\frac{-1}{\beta^{2c}}\right)}{[q_1]} + \frac{[2\alpha\beta(\log t_i)]\left(\frac{-1}{\beta^{2c}}\right)}{(q_2)} + \frac{1}{2} \left(\frac{(q_3)\left(\frac{-2\alpha\beta}{\beta}\right)}{[q_1]^2} + \frac{[q_4][2\alpha\beta(\log t_i)]}{[q_2]^2} \right) \right\} - \left\{ \exp\left[-\left(\frac{t_i^\alpha}{\beta}\right)\right] + \frac{1}{2} \left\{ \frac{\left(\frac{2\alpha\beta}{\beta^2}(\beta+1)\right)}{[q_1]} + \frac{[2\alpha\beta(\log t_i)^2(1+\beta)]}{(q_2)} \right\} + \frac{\left(\frac{-2\alpha\beta}{\beta}\right)\left(\frac{-1}{\beta^{2c}}\right)}{[q_1]} + \frac{[2\alpha\beta(\log t_i)]\left(\frac{-1}{\beta^{2c}}\right)}{(q_2)} + \frac{1}{2} \left(\frac{(q_3)\left(\frac{-2\alpha\beta}{\beta}\right)}{[q_1]^2} + \frac{[q_4][2\alpha\beta(\log t_i)]}{[q_2]^2} \right) \right\}^2 \dots(20)$$

Where

$$q_1 = \left[-\frac{n}{\beta^2} + \frac{2}{\beta^3} \sum_{i=1}^n t_i^\alpha\right], q_2 = \left[\frac{n}{\alpha^2} + \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^2\right], q_3 = \left[-\frac{2n}{\beta^2} + \frac{6}{\beta^4} \sum_{i=1}^n t_i^\alpha\right] \text{ and } q_4 = \left[\frac{2n}{\alpha^2} - \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^2\right]$$

4.3. Lognormal-Inverted Gamma prior under Squared Error Loss Function

The Bayes risk of survival function using Lognormal-Inverted Gamma prior under squared error loss function is

$$R[\hat{S}(t_i)]_{ss} = E \left\{ \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\}^2 \right\} - \left[E \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\} \right]^2 = \frac{\int \int \exp \left(- \left(\frac{t_i^\alpha}{\beta} \right) \right)^2 \pi_2(\alpha, \beta) d\alpha d\beta}{\int \int \pi_2(\alpha, \beta) d\alpha d\beta} - \left\{ \frac{\int \int \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \pi_2(\alpha, \beta) d\alpha d\beta}{\int \int \pi_2(\alpha, \beta) d\alpha d\beta} \right\}^2 \dots(21)$$

$$\rho_1(\alpha, \beta) = -\log \alpha - \frac{(\log \alpha)^2}{2\alpha^2} - (\alpha_1 + 1) \log \beta - \frac{b_1}{\beta}; \quad \rho_1 = \frac{d\rho}{d\beta} = \frac{-(\alpha_1 + 1)}{\beta} + \frac{b_1}{\beta^2}; \quad \rho_2 = \frac{d\rho}{d\alpha} = -\frac{1}{\alpha} - \frac{\log \alpha}{\alpha^2}$$

The Bayes risk of survival function using Lognormal-Inverted Gamma prior under squared error loss function is

$$R[\hat{S}(t_i)]_{ss} = \left[\left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\}^2 + \frac{1}{2} \left\{ \frac{\left[\frac{2u\alpha}{\beta^2}(\alpha+1) \right]}{[\sigma_1]} + \frac{[2u\alpha(\log t_i)^2(1+2\alpha)]}{(\sigma_2)} \right\} + \frac{\left(\frac{-2u\alpha}{\beta} \right) \left(\frac{-[b_1 + t_i \frac{b_1}{\beta}]}{\beta} \right)}{[\sigma_1]} + \frac{(2u\alpha(\log t_i)) \left(\frac{-\frac{1}{\alpha} - \frac{\log \alpha}{\alpha^2}}{(\sigma_2)} \right)}{(\sigma_2)} + \frac{1}{2} \left(\frac{(\sigma_2) \left(\frac{-2u\alpha}{\beta} \right)}{[\sigma_1]^2} + \frac{[2u\alpha(\log t_i)]}{[\sigma_2]^2} \right) \right] - \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] + \frac{1}{2} \left\{ \frac{\left[\frac{2u\alpha}{\beta^2}(\alpha+1) \right]}{[\sigma_1]} + \frac{[2u\alpha(\log t_i)^2(1+2\alpha)]}{(\sigma_2)} \right\} + \frac{\left(\frac{-2u\alpha}{\beta} \right) \left(\frac{-[b_1 + t_i \frac{b_1}{\beta}]}{\beta} \right)}{[\sigma_1]} + \frac{(u\alpha(\log t_i)) \left(\frac{-\frac{1}{\alpha} - \frac{\log \alpha}{\alpha^2}}{(\sigma_2)} \right)}{(\sigma_2)} + \frac{1}{2} \left(\frac{(\sigma_2) \left(\frac{-2u\alpha}{\beta} \right)}{[\sigma_1]^2} + \frac{[2u\alpha(\log t_i)]}{[\sigma_2]^2} \right) \right\}^2 \dots(22)$$

Where

$$q_1 = \left[-\frac{n}{\beta^2} + \frac{1}{\beta^2} \sum_{i=1}^n t_i^\alpha \right], \quad q_2 = \left(\frac{n}{\alpha^2} + \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^2 \right), \quad q_3 = \left(-\frac{2n}{\beta^2} + \frac{b_1}{\beta^2} \sum_{i=1}^n t_i^\alpha \right) \text{ and } q_4 = \left[\frac{2n}{\alpha^2} - \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^2 \right]$$

4.4. Extended Inverse Gamma-Extended Inverse Gamma prior under Squared error loss function

The Bayes risk of survival function using Extended Inverse Gamma-Extended Inverse Gamma prior under Squared error loss function is

$$R[\hat{S}(t_i)]_{ss} = E \left\{ \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\}^2 \right\} - \left[E \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\} \right]^2 = \frac{\int \int \exp \left(- \left(\frac{t_i^\alpha}{\beta} \right) \right)^2 \pi_4(\alpha, \beta) d\alpha d\beta}{\int \int \pi_4(\alpha, \beta) d\alpha d\beta} - \left\{ \frac{\int \int \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \pi_4(\alpha, \beta) d\alpha d\beta}{\int \int \pi_4(\alpha, \beta) d\alpha d\beta} \right\}^2 \dots(23)$$

$$\rho_1(\alpha, \beta) = -2 \left(\alpha_2 + \frac{1}{2} \right) \log \alpha + h_1 \log \left(1 + \frac{1}{\alpha^2} \right) - \frac{b_2}{\alpha^2} - 2 \left(d_2 + \frac{1}{2} \right) \log \beta + h_2 \log \left(1 + \frac{1}{\beta^2} \right) - \frac{c_2}{\beta^2};$$

$$\rho_2 = \frac{d\rho}{d\beta} = \frac{2}{\beta} \left[\frac{c_2}{\beta^2} - \left(\frac{h_2}{\beta^2 + 1} \right) - d_2 - \frac{1}{2} \right]; \quad \rho_1 = \frac{d\rho}{d\alpha} = \frac{2}{\alpha} \left[\frac{b_2}{\alpha^2} - \left(\frac{h_1}{\alpha^2 + 1} \right) - \alpha_2 - \frac{1}{2} \right]$$

The Bayes risk of survival function using Extended Inverse Gamma-Extended Inverse Gamma prior under Squared error loss function is

$$R[S(t_i)]_{ss} = \left[\left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\}^2 + \frac{1}{2} \left\{ \frac{\left[\frac{2u\alpha}{\beta^2}(\alpha+1) \right]}{(\sigma_1)} + \frac{[2u\alpha(\log t_i)^2(1+2\alpha)]}{(\sigma_2)} \right\} + \frac{\left(\frac{-2u\alpha}{\beta} \right) \left[\frac{1}{\beta} \left(\frac{b_2}{\alpha^2} - \left(\frac{h_2}{\alpha^2 + 1} \right) - d_2 - \frac{1}{2} \right) \right]}{(\sigma_1)} + \frac{(2u\alpha(\log t_i)) \left[\frac{1}{\alpha} \left(\frac{b_1}{\alpha^2} - \left(\frac{h_1}{\alpha^2 + 1} \right) - \alpha_2 - \frac{1}{2} \right) \right]}{(\sigma_2)} \right] + \frac{1}{2} \left(\frac{(\sigma_2) \left(\frac{-2u\alpha}{\beta} \right)}{(\sigma_1)^2} + \frac{[2u\alpha(\log t_i)]}{[\sigma_2]^2} \right) \right] - \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] + \frac{1}{2} \left\{ \frac{\left[\frac{2u\alpha}{\beta^2}(\alpha+1) \right]}{(\sigma_1)} + \frac{[2u\alpha(\log t_i)^2(1+2\alpha)]}{(\sigma_2)} \right\} + \frac{\left(\frac{-2u\alpha}{\beta} \right) \left[\frac{1}{\beta} \left(\frac{b_2}{\alpha^2} - \left(\frac{h_2}{\alpha^2 + 1} \right) - d_2 - \frac{1}{2} \right) \right]}{(\sigma_1)} + \frac{(u\alpha(\log t_i)) \left[\frac{1}{\alpha} \left(\frac{b_1}{\alpha^2} - \left(\frac{h_1}{\alpha^2 + 1} \right) - \alpha_2 - \frac{1}{2} \right) \right]}{(\sigma_2)} \right] + \frac{1}{2} \left(\frac{(\sigma_2) \left(\frac{-2u\alpha}{\beta} \right)}{(\sigma_1)^2} + \frac{[2u\alpha(\log t_i)]}{[\sigma_2]^2} \right) \right\}^2 \dots(24)$$

Where

$$q_1 = \left[-\frac{n}{\beta^2} + \frac{1}{\beta^2} \sum_{i=1}^n t_i^2\right], q_2 = \left(\frac{n}{\beta^2} + \frac{1}{\beta} \sum_{i=1}^n t_i^2 (\log t_i)^2\right), q_3 = \left(-\frac{2n}{\beta^2} + \frac{6}{\beta^4} \sum_{i=1}^n t_i^4\right) \text{ and } q_4 = \left[\frac{2n}{\beta^2} - \frac{1}{\beta} \sum_{i=1}^n t_i^2 (\log t_i)^2\right]$$

4.5. Jeffrey’s prior under Linear Exponential Loss Function

The Bayes risk of survival function using Jeffrey’s prior under Linex loss function is

$$R[\hat{S}(t_i)]_{JL} = \log \left\{ E \left(e^{-m \left[\exp \left[-\left(\frac{t_i^2}{\beta} \right) \right] \right]} \right) \right\} + m E \left\{ \exp \left[-\left(\frac{t_i^2}{\beta} \right) \right] \right\} =$$

$$\log \left\{ \frac{\int \int e^{-m \left[\exp \left[-\left(\frac{t_i^2}{\beta} \right) \right] \right]} \pi_1(\alpha, \beta) d\beta d\alpha}{\int \int \pi_1(\alpha, \beta) d\beta d\alpha} \right\} + m \left\{ \frac{\int \int \exp \left[-\left(\frac{t_i^2}{\beta} \right) \right] \pi_1(\alpha, \beta) d\beta d\alpha}{\int \int \pi_1(\alpha, \beta) d\beta d\alpha} \right\} \quad \dots(25)$$

Let $u = e^{-m \left[\exp \left[-\left(\frac{t_i^2}{\beta} \right) \right] \right]}$ and $q = \exp \left[-\left(\frac{t_i^2}{\beta} \right) \right]$,

$$u_1 = \frac{du}{d\beta} = -mq u \left(\frac{t_i^2}{\beta^2} \right); \quad u_{11} = \frac{d^2u}{d\beta^2} = mqu \left(\frac{t_i^2}{\beta^4} \right) (mq t_i^2 - t_i^2 + 2\beta);$$

$$u_2 = \frac{du}{d\alpha} = mqu \left(\frac{t_i^2 \log t_i}{\beta} \right); \quad u_{22} = \frac{d^2u}{d\alpha^2} = mqu \left(\frac{t_i^2 (\log t_i)^2}{\beta} \right) \left(1 + mq \frac{t_i^2}{\beta} - \frac{t_i^2}{\beta} \right)$$

The Bayes risk of survival function using Jeffrey’s prior under Linex loss function is

$$R[\hat{S}(t_i)]_{JL} = \left[\log \left\{ e^{-m \left[\exp \left[-\left(\frac{t_i^2}{\beta} \right) \right] \right]} + \frac{1}{2} \left\{ \frac{mq u \left(\frac{t_i^2}{\beta^2} \right) (mq t_i^2 - t_i^2 + 2\beta)}{(\sigma_2)} + \frac{mq u \left(\frac{t_i^2 (\log t_i)^2}{\beta} \right) (1 + mq \frac{t_i^2}{\beta} - \frac{t_i^2}{\beta})}{(\sigma_2)} \right\} \right\} + \frac{(-mq u \left(\frac{t_i^2}{\beta^2} \right) \left(\frac{-1}{\beta} \right))}{(\sigma_2)} + \right.$$

$$\left. \frac{(-mq u \left(\frac{t_i^2 \log t_i}{\beta} \right) \left(\frac{-1}{\beta} \right))}{(\sigma_2)} + \frac{1}{2} \left(\frac{(\sigma_2) (-mq u \left(\frac{t_i^2}{\beta^2} \right))}{(\sigma_2)^2} + \frac{[\sigma_2] [mq u \left(\frac{t_i^2 \log t_i}{\beta} \right)]}{[\sigma_2]^2} \right) \right\} + \left[m \left\{ \exp \left[-\left(\frac{t_i^2}{\beta} \right) \right] + \frac{1}{2} \left\{ \frac{mq u (\sigma_2 + 1)}{(\sigma_2)} + \frac{2mq (\log t_i)^2 (1 + \sigma_2)}{(\sigma_2)} \right\} + \frac{(-mq) \left(\frac{-1}{\beta} \right)}{(\sigma_2)} + \right. \right.$$

$$\left. \left. \frac{(mq (\log t_i)) \left(\frac{-1}{\beta} \right)}{(\sigma_2)} + \frac{1}{2} \left(\frac{(\sigma_2) \left(\frac{-mq}{\beta} \right)}{(\sigma_2)^2} + \frac{[\sigma_2] [mq (\log t_i)]}{[\sigma_2]^2} \right) \right\} \right] \quad \dots(26)$$

Where

$$q_1 = \left[-\frac{n}{\beta^2} + \frac{1}{\beta^2} \sum_{i=1}^n t_i^2\right], q_2 = \left(\frac{n}{\beta^2} + \frac{1}{\beta} \sum_{i=1}^n t_i^2 (\log t_i)^2\right), q_3 = \left(-\frac{2n}{\beta^2} + \frac{6}{\beta^4} \sum_{i=1}^n t_i^4\right) \text{ and } q_4 = \left[\frac{2n}{\beta^2} - \frac{1}{\beta} \sum_{i=1}^n t_i^2 (\log t_i)^2\right]$$

4.6. Extension of Jeffrey’s prior under Linear Exponential Loss Function

The Bayes risk of survival function using Extension of Jeffrey’s prior under Linex loss function is

$$R[\hat{S}(t_i)]_{JL} = \log \left\{ E \left(e^{-m \left[\exp \left[-\left(\frac{t_i^2}{\beta} \right) \right] \right]} \right) \right\} + m E \left\{ \exp \left[-\left(\frac{t_i^2}{\beta} \right) \right] \right\} =$$

$$\log \left\{ \frac{\int \int e^{-m \left[\exp \left[-\left(\frac{t_i^2}{\beta} \right) \right] \right]} \pi_1(\alpha, \beta) d\beta d\alpha}{\int \int \pi_1(\alpha, \beta) d\beta d\alpha} \right\} + m \left\{ \frac{\int \int \exp \left[-\left(\frac{t_i^2}{\beta} \right) \right] \pi_1(\alpha, \beta) d\beta d\alpha}{\int \int \pi_1(\alpha, \beta) d\beta d\alpha} \right\} \quad \dots (27)$$

$$\begin{aligned}
 &= \left[\log \left\{ e^{-m \left\{ \exp \left[-\left(\frac{t_i^c}{\beta} \right) \right] \right\}} + \frac{1}{2} \left\{ \frac{[mq_u \left(\frac{t_i^c}{\beta} \right) (mq_{t_i^c} - t_i^c + 2\beta)]}{(\sigma_1)} + \frac{[mq_u \left(\frac{t_i^c (\log t_i)^2}{\beta} \right) (1 + mq_{t_i^c} - t_i^c)]}{(\sigma_2)} \right\} + \frac{(-mq_u \left(\frac{t_i^c}{\beta} \right)) \left(\frac{-1}{\sigma_1} \right)}{(\sigma_1)} + \right. \\
 &\left. \frac{(mq_u \left(\frac{t_i^c \log t_i}{\beta} \right)) \left(\frac{-1}{\sigma_2} \right)}{(\sigma_2)} + \frac{1}{2} \left(\frac{(\sigma_1) (-mq_u \left(\frac{t_i^c}{\beta} \right))}{(\sigma_1)^2} + \frac{[\sigma_1] [mq_u \left(\frac{t_i^c \log t_i}{\beta} \right)]}{[\sigma_1]^2} \right) \right\} + \left[m \left\{ \exp \left[-\left(\frac{t_i^c}{\beta} \right) \right] + \frac{1}{2} \left\{ \frac{[u_c (\log t_i)^2 (1+c)]}{(\sigma_1)} + \frac{[2u_c (\log t_i)^2 (1+c)]}{(\sigma_2)} \right\} + \right. \right. \\
 &\left. \left. \frac{(-u_c) \left(\frac{-1}{\sigma_1} \right)}{(\sigma_1)} + \frac{(u_c (\log t_i)) \left(\frac{-1}{\sigma_2} \right)}{(\sigma_2)} + \frac{1}{2} \left(\frac{(\sigma_1) \left(\frac{-u_c}{\sigma_1} \right)}{(\sigma_1)^2} + \frac{[\sigma_1] [u_c (\log t_i)]}{[\sigma_1]^2} \right) \right\} \right] \quad \dots(28)
 \end{aligned}$$

Where

$$q_1 = \left[-\frac{n}{\beta^2} + \frac{2}{\beta^2} \sum_{i=1}^n t_i^c \right], q_2 = \left(\frac{n}{\sigma_1^2} + \frac{1}{\beta} \sum_{i=1}^n t_i^c (\log t_i)^2 \right), q_3 = \left(-\frac{2n}{\beta^2} + \frac{6}{\beta^2} \sum_{i=1}^n t_i^c \right) \text{ and } q_4 = \left[\frac{2n}{\sigma_2^2} - \frac{1}{\beta} \sum_{i=1}^n t_i^c (\log t_i)^2 \right]$$

4.7. Lognormal-Inverted Gamma prior under Linear Exponential Loss Function

The Bayes risk of survival function using Lognormal-Inverted Gamma prior under Linex loss function is

$$\begin{aligned}
 R[\hat{S}(t_i)]_{IL} &= \log \left\{ E \left(e^{-m \left\{ \exp \left[-\left(\frac{t_i^c}{\beta} \right) \right] \right\}} \right) \right\} + m E \left\{ \exp \left[-\left(\frac{t_i^c}{\beta} \right) \right] \right\} = \\
 &\log \left\{ \frac{\int \int e^{-m \left\{ \exp \left[-\left(\frac{t_i^c}{\beta} \right) \right] \right\}} \pi_2(\alpha, \beta) d\beta d\alpha}{\int \int \pi_2(\alpha, \beta) d\beta d\alpha} \right\} + m \left\{ \frac{\int \int \exp \left[-\left(\frac{t_i^c}{\beta} \right) \right] \pi_2(\alpha, \beta) d\beta d\alpha}{\int \int \pi_2(\alpha, \beta) d\beta d\alpha} \right\} \quad \dots(29)
 \end{aligned}$$

The Bayes risk of survival function using Lognormal-Inverted Gamma prior under Linex loss function is

$$\begin{aligned}
 R[\hat{S}(t_i)]_{IL} &= \left[\log \left\{ e^{-m \left\{ \exp \left[-\left(\frac{t_i^c}{\beta} \right) \right] \right\}} + \frac{1}{2} \left\{ \frac{[mq_u \left(\frac{t_i^c}{\beta} \right) (mq_{t_i^c} - t_i^c + 2\beta)]}{(\sigma_1)} + \frac{[mq_u \left(\frac{t_i^c (\log t_i)^2}{\beta} \right) (1 + mq_{t_i^c} - t_i^c)]}{(\sigma_2)} \right\} + \frac{(-mq_u \left(\frac{t_i^c}{\beta} \right)) \left(\frac{-1}{\sigma_1} \right) \left(\frac{t_i^c}{\beta} \right)}{(\sigma_1)} + \right. \\
 &\left. \frac{(mq_u \left(\frac{t_i^c \log t_i}{\beta} \right)) \left(\frac{-1}{\sigma_2} \right) \left(\frac{-1}{\sigma_2} \right)}{(\sigma_2)} + \frac{1}{2} \left(\frac{(\sigma_1) (-mq_u \left(\frac{t_i^c}{\beta} \right))}{(\sigma_1)^2} + \frac{[\sigma_1] [mq_u \left(\frac{t_i^c \log t_i}{\beta} \right)]}{[\sigma_1]^2} \right) \right\} + \left[m \left\{ \exp \left[-\left(\frac{t_i^c}{\beta} \right) \right] + \frac{1}{2} \left\{ \frac{[u_c (\log t_i)^2 (1+c)]}{(\sigma_1)} + \frac{[2u_c (\log t_i)^2 (1+c)]}{(\sigma_2)} \right\} + \right. \right. \\
 &\left. \left. \frac{(-u_c) \left(\frac{-1}{\sigma_1} \right) \left(\frac{t_i^c}{\beta} \right)}{(\sigma_1)} + \frac{(u_c (\log t_i)) \left(\frac{-1}{\sigma_2} \right) \left(\frac{-1}{\sigma_2} \right)}{(\sigma_2)} + \frac{1}{2} \left(\frac{(\sigma_1) \left(\frac{-u_c}{\sigma_1} \right)}{(\sigma_1)^2} + \frac{[\sigma_1] [u_c (\log t_i)]}{[\sigma_1]^2} \right) \right\} \right] \quad \dots(30)
 \end{aligned}$$

Where

$$q_1 = \left[-\frac{n}{\beta^2} + \frac{2}{\beta^2} \sum_{i=1}^n t_i^c \right], q_2 = \left(\frac{n}{\sigma_1^2} + \frac{1}{\beta} \sum_{i=1}^n t_i^c (\log t_i)^2 \right), q_3 = \left(-\frac{2n}{\beta^2} + \frac{6}{\beta^2} \sum_{i=1}^n t_i^c \right) \text{ and } q_4 = \left[\frac{2n}{\sigma_2^2} - \frac{1}{\beta} \sum_{i=1}^n t_i^c (\log t_i)^2 \right]$$

4.8. Extended Inverse Gamma-Extended Inverse Gamma prior under Linear Exponential Loss Function

The Bayes risk of survival function using Extended Inverse Gamma-Extended Inverse Gamma prior under Linear exponential loss function is

$$R[\hat{S}(t_i)]_{\text{EL}} = \log \left\{ E \left(e^{-m \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\}} \right) \right\} + m E \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\} =$$

$$\log \left\{ \frac{\int \int e^{-m \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\}} \pi_4(\alpha, \beta) d\alpha d\beta}{\int \int \pi_4(\alpha, \beta) d\alpha d\beta} \right\} + m \left\{ \frac{\int \int \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \pi_4(\alpha, \beta) d\alpha d\beta}{\int \int \pi_4(\alpha, \beta) d\alpha d\beta} \right\}$$

...(31)

The Bayes risk of survival function using Extended Inverse Gamma-Extended Inverse Gamma prior under Linear loss function is

$$R[S(t_i)]_{\text{L}} = \left[\log \left\{ e^{-m \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\}} + \frac{1}{2} \left\{ \frac{[m \alpha u \left(\frac{t_i^\alpha}{\beta} \right) (m \alpha t_i^\alpha - t_i^\alpha + 2\beta)]}{(\sigma_1)} + \frac{[m \alpha u \left(\frac{t_i^\alpha}{\beta} \right) (1 + m \alpha \frac{t_i^\alpha - t_i^\alpha}{\beta})]}{(\sigma_2)} \right\} + \frac{(-m \alpha u \left(\frac{t_i^\alpha}{\beta} \right) \left(\frac{t_i^\alpha}{\beta} - \left(\frac{t_i^\alpha}{\beta} \right) - d_1 - \frac{1}{2} \right])}{(\sigma_2)} + \right.$$

$$\left. \frac{(m \alpha u \left(\frac{t_i^\alpha}{\beta} \right) \left(\frac{t_i^\alpha}{\beta} - \left(\frac{t_i^\alpha}{\beta} \right) - d_1 - \frac{1}{2} \right])}{(\sigma_2)} + \frac{1}{2} \left\{ \frac{(\sigma_2) \left(-m \alpha u \left(\frac{t_i^\alpha}{\beta} \right) \right)}{(\sigma_1)^2} + \frac{[\sigma_1] [m \alpha u \left(\frac{t_i^\alpha}{\beta} \right)]}{[\sigma_2]^2} \right\} \right\} + \left[m \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] + \frac{1}{2} \left\{ \frac{[u \alpha (\alpha + 2)]}{(\sigma_1)} + \frac{[2 u \alpha (\log t_i)^2 (1 + \alpha)]}{(\sigma_2)} + \right. \right.$$

$$\left. \left. \frac{(\alpha u \left(\frac{t_i^\alpha}{\beta} \right) \left(\frac{t_i^\alpha}{\beta} - \left(\frac{t_i^\alpha}{\beta} \right) - d_1 - \frac{1}{2} \right])}{(\sigma_1)} + \frac{(u \alpha (\log t_i)) \left(\frac{t_i^\alpha}{\beta} - \left(\frac{t_i^\alpha}{\beta} \right) - d_1 - \frac{1}{2} \right)]}{(\sigma_2)} + \frac{1}{2} \left\{ \frac{(\sigma_2) \left(-\alpha u \right)}{(\sigma_1)^2} + \frac{[\sigma_1] [u \alpha (\log t_i)]}{[\sigma_2]^2} \right\} \right\} \right]$$

...(32)

Where

$$q_1 = \left[-\frac{\alpha}{\beta^2} + \frac{1}{\beta^2} \sum_{i=1}^n t_i^\alpha \right], q_2 = \left(\frac{\alpha}{\beta} + \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^2 \right), q_3 = \left(-\frac{2\alpha}{\beta^2} + \frac{6}{\beta^2} \sum_{i=1}^n t_i^\alpha \right) \text{ and } q_4 = \left[\frac{2\alpha}{\beta^2} - \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^2 \right]$$

4.9. Jeffrey’s prior under General Entropy Loss Function

The Bayes risk of survival function using Jeffrey’s prior under General entropy loss function is

$$R[\hat{S}(t_i)]_{\text{GE}} = k E \left[\log \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\} \right] - \log E \left\{ \left[\exp \left(- \left(\frac{t_i^\alpha}{\beta} \right) \right) \right]^{-k} \right\} =$$

$$k \left[\frac{\int \int \log \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\} \pi_4(\alpha, \beta) d\alpha d\beta}{\int \int \pi_4(\alpha, \beta) d\alpha d\beta} \right] - \log \left\{ \frac{\int \int \left[\exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right]^{-k} \pi_4(\alpha, \beta) d\alpha d\beta}{\int \int \pi_4(\alpha, \beta) d\alpha d\beta} \right\}$$

...(33)

Let $u = \log \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\} = - \left(\frac{t_i^\alpha}{\beta} \right)$; $u_1 = \frac{du}{d\beta} = \frac{-\alpha}{\beta}$; $u_{11} = \frac{d^2u}{d\beta^2} = \frac{2\alpha}{\beta^2}$; $u_2 = \frac{du}{d\alpha} = u(\log t_i)$; $u_{22} = \frac{d^2u}{d\alpha^2} = u(\log t_i)^2$

$$u = \left[\exp \left(- \left(\frac{t_i^\alpha}{\beta} \right) \right) \right]^{-k}$$

$$u_1 = \frac{du}{d\beta} = \frac{k u \alpha}{\beta}$$
 ; $u_{11} = \frac{d^2u}{d\beta^2} = \frac{k u \alpha}{\beta^2} (k\alpha - 2)$; $u_2 = \frac{du}{d\alpha} = -k u e(\log t_i)$; $u_{22} = \frac{d^2u}{d\alpha^2} = k u e(\log t_i)^2 (k\alpha - 1)$

The Bayes risk of survival function using Jeffrey's prior under General entropy loss function is

$$R[\hat{S}(t_i)]_{gc} = k \left\{ \log \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\} + \frac{1}{z} \left\{ \frac{\left(\frac{t_i^\alpha}{\beta} \right)}{\left(\sigma_1 \right)} + \frac{[u(\log t_i)]^2}{\left(\sigma_2 \right)} + \frac{\left(\frac{-t_i^\alpha}{\beta} \right)}{\left(\sigma_1 \right)} + \frac{[u(\log t_i)] \left(\frac{-t_i^\alpha}{\beta} \right)}{\left(\sigma_2 \right)} + \frac{1}{z} \left(\frac{[\sigma_1] \left(\frac{-t_i^\alpha}{\beta} \right)}{\left(\sigma_1 \right)^2} + \frac{[\sigma_2] [u(\log t_i)]}{[\sigma_2]^2} \right) \right\} - \log \left\{ \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\}^{-k} + \frac{1}{z} \left\{ \frac{[k u \alpha (\log t_i)^2 (k z - 1)]}{\left(\sigma_1 \right)} + \frac{[k u \alpha (\log t_i)]^2 (k z - 1)]}{\left(\sigma_2 \right)} + \frac{\left(\frac{k u \alpha}{\beta} \right) \left(\frac{-1}{\beta} \right)}{\left(\sigma_1 \right)} + \frac{(-k u \alpha (\log t_i)) \left(\frac{-1}{\beta} \right)}{\left(\sigma_2 \right)} + \frac{1}{z} \left(\frac{[\sigma_1] \left(\frac{k u \alpha}{\beta} \right)}{\left(\sigma_1 \right)^2} + \frac{[\sigma_2] [-k u \alpha (\log t_i)]}{[\sigma_2]^2} \right) \right\} \right\} \dots(34)$$

Where

$$q_1 = \left[-\frac{z}{\beta^2} + \frac{z}{\beta^2} \sum_{i=1}^n t_i^\alpha \right], q_2 = \left(\frac{z}{\sigma_1^2} + \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^2 \right), q_3 = \left(-\frac{z}{\beta^2} + \frac{z}{\beta^2} \sum_{i=1}^n t_i^\alpha \right) \text{ and } q_4 = \left[\frac{z}{\sigma_1^2} - \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^2 \right]$$

4.10.Extension of Jeffrey's prior under General Entropy Loss Function

The Bayes risk of survival function using Extension of Jeffrey's prior under General entropy loss function is

$$R[\hat{S}(t_i)]_{gc} = k E \left[\log \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\} \right] - \log E \left\{ \left[\exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right]^{-k} \right\} = k \left[\frac{\int \int \log \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\} \pi_2(\alpha, \beta) d\beta d\alpha}{\int \int \pi_2(\alpha, \beta) d\beta d\alpha} - \log \left\{ \frac{\int \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\}^{-k} \pi_2(\alpha, \beta) d\beta d\alpha}{\int \int \pi_2(\alpha, \beta) d\beta d\alpha} \right\} \right] \dots(35)$$

The Bayes risk of survival function under General entropy loss function is

$$R[\hat{S}(t_i)]_{gc} = k \left\{ \log \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\} + \frac{1}{z} \left\{ \frac{\left(\frac{t_i^\alpha}{\beta} \right)}{\left(\sigma_1 \right)} + \frac{[u(\log t_i)]^2}{\left(\sigma_2 \right)} + \frac{\left(\frac{-t_i^\alpha}{\beta} \right)}{\left(\sigma_1 \right)} + \frac{[u(\log t_i)] \left(\frac{-t_i^\alpha}{\beta} \right)}{\left(\sigma_2 \right)} + \frac{1}{z} \left(\frac{[\sigma_1] \left(\frac{-t_i^\alpha}{\beta} \right)}{\left(\sigma_1 \right)^2} + \frac{[\sigma_2] [u(\log t_i)]}{[\sigma_2]^2} \right) \right\} - \log \left\{ \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\}^{-k} + \frac{1}{z} \left\{ \frac{[k u \alpha (\log t_i)^2 (k z - 1)]}{\left(\sigma_1 \right)} + \frac{[k u \alpha (\log t_i)]^2 (k z - 1)]}{\left(\sigma_2 \right)} + \frac{\left(\frac{k u \alpha}{\beta} \right) \left(\frac{-1}{\beta} \right)}{\left(\sigma_1 \right)} + \frac{(-k u \alpha (\log t_i)) \left(\frac{-1}{\beta} \right)}{\left(\sigma_2 \right)} + \frac{1}{z} \left(\frac{[\sigma_1] \left(\frac{k u \alpha}{\beta} \right)}{\left(\sigma_1 \right)^2} + \frac{[\sigma_2] [-k u \alpha (\log t_i)]}{[\sigma_2]^2} \right) \right\} \right\} \dots(36)$$

Where

$$q_1 = \left[-\frac{z}{\beta^2} + \frac{z}{\beta^2} \sum_{i=1}^n t_i^\alpha \right], q_2 = \left(\frac{z}{\sigma_1^2} + \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^2 \right), q_3 = \left(-\frac{z}{\beta^2} + \frac{z}{\beta^2} \sum_{i=1}^n t_i^\alpha \right) \text{ and } q_4 = \left[\frac{z}{\sigma_1^2} - \frac{1}{\beta} \sum_{i=1}^n t_i^\alpha (\log t_i)^2 \right]$$

4.11. Lognormal-Inverted Gamma prior under General Entropy Loss Function

The Bayes risk of survival function using Lognormal-Inverted Gamma prior under General entropy loss function is

$$R[\hat{S}(t_i)]_{gc} = k E \left[\log \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\} \right] - \log E \left\{ \left[\exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right]^{-k} \right\} = k \left[\frac{\int \int \log \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\} \pi_2(\alpha, \beta) d\beta d\alpha}{\int \int \pi_2(\alpha, \beta) d\beta d\alpha} - \log \left\{ \frac{\int \left\{ \exp \left[- \left(\frac{t_i^\alpha}{\beta} \right) \right] \right\}^{-k} \pi_2(\alpha, \beta) d\beta d\alpha}{\int \int \pi_2(\alpha, \beta) d\beta d\alpha} \right\} \right] \dots (37)$$

The Bayes risk of survival function using Lognormal-Inverted Gamma prior under General entropy loss function is

$$R[\hat{S}(t_i)]_{GC} = k \left\{ \log \left\{ \exp \left[- \left(\frac{t_i^{\alpha}}{\beta} \right) \right] \right\} + \frac{1}{2} \left\{ \frac{\left[\frac{2u}{\sigma_1^2} \right] + \frac{[u(\log t_i)^2]}{(\sigma_2)} \right\} + \frac{\left(\frac{u}{\beta} \right) \left(- \frac{(\alpha_1 + t_i)}{\beta} + \frac{b_1}{\beta^2} \right)}{(\sigma_1)} + \frac{[u(\log t_i)] \left(- \frac{1}{\alpha} - \frac{\log t_i}{\alpha \sigma_1^2} \right)}{(\sigma_2)} + \frac{1}{2} \left(\frac{[\sigma_1] \left(\frac{u}{\beta} \right) + \frac{[\sigma_4] [u(\log t_i)]}{[\sigma_2]^2} \right) \right\} - \log \left\{ \exp \left[- \left(\frac{t_i^{\alpha}}{\beta} \right) \right]^{-k} + \frac{1}{2} \left\{ \frac{[k u \alpha (k \alpha - 2)]}{\sigma_1^2} + \frac{[k u \alpha (\log t_i)^2 (k \alpha - 1)]}{(\sigma_2)} \right\} + \frac{\left(\frac{k u \alpha}{\beta} \right) \left(- \frac{(\alpha_1 + t_i)}{\beta} + \frac{b_1}{\beta^2} \right)}{(\sigma_1)} + \frac{(-k u \alpha (\log t_i)) \left(- \frac{1}{\alpha} - \frac{\log t_i}{\alpha \sigma_1^2} \right)}{(\sigma_2)} + \frac{1}{2} \left(\frac{[\sigma_1] \left(\frac{k u \alpha}{\beta} \right) + \frac{[\sigma_4] [-k u \alpha (\log t_i)]}{[\sigma_2]^2} \right) \right\} \right\} \dots(38)$$

Where

$$q_1 = \left[- \frac{\alpha}{\beta^2} + \frac{1}{\beta^2} \sum_{i=1}^n t_i^{\alpha} \right], q_2 = \left(\frac{\alpha}{\sigma_1^2} + \frac{1}{\beta} \sum_{i=1}^n t_i^{\alpha} (\log t_i)^2 \right), q_3 = \left(- \frac{2u}{\beta^2} + \frac{u}{\beta^2} \sum_{i=1}^n t_i^{\alpha} \right) \text{ and } q_4 = \left[\frac{2u}{\sigma_2^2} - \frac{1}{\beta} \sum_{i=1}^n t_i^{\alpha} (\log t_i)^2 \right]$$

4.12. Extended Inverse Gamma - Extended Inverse Gamma prior under General Entropy Loss Function

The Bayes risk of survival function using Extended Inverse Gamma - Extended Inverse Gamma prior under General entropy loss function is

$$R[\hat{S}(t_i)]_{GC} = k E \left[\log \left\{ \exp \left[- \left(\frac{t_i^{\alpha}}{\beta} \right) \right] \right\} \right] - \log E \left\{ \left[\exp \left(- \left(\frac{t_i^{\alpha}}{\beta} \right) \right) \right]^{-k} \right\} = k \left[\frac{\int \int \log \left\{ \exp \left[- \left(\frac{t_i^{\alpha}}{\beta} \right) \right] \right\} \pi_4(\alpha, \beta) d\beta d\alpha}{\int \int \pi_4(\alpha, \beta) d\beta d\alpha} \right] - \log \left\{ \frac{\int \left[\exp \left[- \left(\frac{t_i^{\alpha}}{\beta} \right) \right] \right]^{-k} \pi_4(\alpha, \beta) d\beta d\alpha}{\int \int \pi_4(\alpha, \beta) d\beta d\alpha} \right\} \right] \dots(39)$$

The Bayes risk of survival function using Extended Inverse Gamma - Extended Inverse Gamma prior under General entropy loss function is

$$R[\hat{S}(t_i)]_{GC} = k \left\{ \log \left\{ \exp \left[- \left(\frac{t_i^{\alpha}}{\beta} \right) \right] \right\} + \frac{1}{2} \left\{ \frac{\left[\frac{2u}{\sigma_1^2} \right] + \frac{[u(\log t_i)^2]}{(\sigma_2)} \right\} + \frac{\left(\frac{u}{\beta} \right) \left(\frac{2}{\sigma_1} \left[\frac{2}{\sigma_1^2} - \left(\frac{b_1}{\sigma_1^2 + t_i} \right) - d_1 - \frac{1}{2} \right] \right)}{(\sigma_1)} + \frac{[u(\log t_i)] \left(\frac{2}{\sigma_2} \left[\frac{2}{\sigma_2^2} - \left(\frac{b_1}{\sigma_2^2 + t_i} \right) - d_2 - \frac{1}{2} \right] \right)}{(\sigma_2)} + \frac{1}{2} \left(\frac{[\sigma_1] \left(\frac{u}{\beta} \right) + \frac{[\sigma_4] [u(\log t_i)]}{[\sigma_2]^2} \right) \right\} - \log \left\{ \exp \left[- \left(\frac{t_i^{\alpha}}{\beta} \right) \right]^{-k} + \frac{1}{2} \left\{ \frac{[k u \alpha (k \alpha - 2)]}{\sigma_1^2} + \frac{[k u \alpha (\log t_i)^2 (k \alpha - 1)]}{(\sigma_2)} \right\} + \frac{\left(\frac{k u \alpha}{\beta} \right) \left(\frac{2}{\sigma_1} \left[\frac{2}{\sigma_1^2} - \left(\frac{b_1}{\sigma_1^2 + t_i} \right) - d_1 - \frac{1}{2} \right] \right)}{(\sigma_1)} + \frac{(-k u \alpha (\log t_i)) \left(\frac{2}{\sigma_2} \left[\frac{2}{\sigma_2^2} - \left(\frac{b_1}{\sigma_2^2 + t_i} \right) - d_2 - \frac{1}{2} \right] \right)}{(\sigma_2)} + \frac{1}{2} \left(\frac{[\sigma_1] \left(\frac{k u \alpha}{\beta} \right) + \frac{[\sigma_4] [-k u \alpha (\log t_i)]}{[\sigma_2]^2} \right) \right\} \right\} \dots(40)$$

Where

$$q_1 = \left[- \frac{\alpha}{\beta^2} + \frac{1}{\beta^2} \sum_{i=1}^n t_i^{\alpha} \right], q_2 = \left(\frac{\alpha}{\sigma_1^2} + \frac{1}{\beta} \sum_{i=1}^n t_i^{\alpha} (\log t_i)^2 \right), q_3 = \left(- \frac{2u}{\beta^2} + \frac{u}{\beta^2} \sum_{i=1}^n t_i^{\alpha} \right) \text{ and } q_4 = \left[\frac{2u}{\sigma_2^2} - \frac{1}{\beta} \sum_{i=1}^n t_i^{\alpha} (\log t_i)^2 \right]$$

V. SIMULATION AUDIT

In this system, we picked a mixture season of n = 25, 50, and 100 to test that we forgot to manipulate anything close to anything, medium, or epic dataset. The once more danger of wonderful superb craftsmanship is endeavored up for Weibull spreading with halting the usage of non-illuminating and edifying priors, for example, Jeffrey's, Progress of Jeffrey's, Lognormal-Turned Gamma and Extended Change Gamma-Expanded Talk Gamma using squared goof problem monster stones, Linux episode cutoff, and General entropy torture imaginative aftereffects and incidental consequences. The endpoints' expansive physical activities are

determined to be = 0.5, 1.5, and = 0.8, 1.2. These are iterated (N) on specific video games and the norm, damaged down superb inventive appearances for the existing and then premise unquestionably. The proposed rectangular abusing (MSE) is depicted as

$$MSE = \frac{1}{N} \sum_{j=1}^N \left\{ \frac{1}{n} \sum_{i=1}^n [R_j(\hat{S}(t_i)) - R_j(S(t_i))]^2 \right\}$$

The MSE for the estimated survival function of Weibull distribution with two parameters were obtained and presented in Table -1.

Table-1: MSE of Bayes risk of Survival Function under Jeffrey’s prior

The MSE of Bayes risk of Weibull survival functions were obtained and presented in Table -1.

n	β	α	R(S(t)) _{BB}	R(S(t)) _{BB}	R(S(t)) _{BB}	R(S(t)) _{BB}	R(S(t)) _{BB}	R(S(t)) _{BB}	R(S(t)) _{BB}	R(S(t)) _{BB}	R(S(t)) _{BB}	
				m=k=0.6	m=k=-0.6	m=k=1.6	m=k=-1.6					
25	0.	0.	3.17201	1.02210	6.32100	1.03204	6.70658	5.10684	2.20604	5.24037	2.51600	
		5	8	5e-09	3e-10	4e-08	6e-10	6e-08	2e-09	5e-06	1e-09	1e-06
		1.	3.17537	1.02319	6.32107	1.03325	6.70661	5.11220	2.20636	5.24590	2.51630	
		2	8e-09	7e-10	0e-08	2e-10	3e-08	5e-09	8e-06	4e-09	2e-06	
	1.	0.	1.17595	3.82423	5.90193	3.79422	5.81930	1.94240	1.81164	1.90141	1.76003	
		5	8	5e-09	5e-11	9e-07	1e-11	0e-07	0e-09	0e-05	1e-09	2e-05
		1.	1.18009	3.83738	5.95706	3.80761	5.87278	1.94939	1.82697	1.90794	1.77467	
		2	9e-09	1e-11	7e-07	5e-11	5e-07	0e-09	6e-05	2e-09	9e-05	
50	0.	0.	6.30269	2.04439	2.52624	2.03761	2.55346	1.03219	8.69666	1.02357	8.87914	
		5	8	2e-10	2e-11	7e-08	0e-11	5e-08	4e-09	1e-07	8e-09	9e-07
		1.	6.30226	2.04436	2.52537	2.03720	2.55270	1.03214	8.69316	1.02348	8.87567	
		2	9e-10	7e-11	6e-08	0e-11	1e-08	0e-09	1e-07	4e-09	2e-07	
	1.	0.	4.39167	1.43667	9.77179	1.40770	9.47628	7.35748	3.13111	6.96807	2.95276	
		5	8	7e-10	5e-11	8e-08	6e-11	7e-08	7e-10	5e-06	8e-10	5e-06
		1.	4.39333	1.43730	9.85198	1.40827	9.55566	7.36004	3.16456	6.97072	2.98550	
		2	3e-10	5e-11	3e-08	4e-11	9e-08	8e-10	2e-06	8e-10	3e-06	
100	0.	0.	4.98434	1.61620	2.95394	1.62102	3.17818	8.12700	6.95654	8.17656	7.89152	
		5	8	1e-11	1e-12	1e-09	9e-12	6e-09	0e-11	8e-07	6e-11	6e-07
		1.	4.98586	1.61726	2.95405	1.62032	3.17837	8.12878	6.95712	8.17899	7.89224	
		2	7e-11	4e-12	1e-09	7e-12	5e-09	4e-11	9e-07	1e-11	4e-07	
	1.	0.	1.92385	6.26607	6.81802	6.26202	6.77262	3.16019	1.57927	3.13729	1.52981	
		5	8	8e-11	1e-13	1e-09	1e-13	4e-09	6e-11	6e-06	2e-11	3e-06
		1.	2.20789	7.18317	7.65901	6.27891	7.07683	3.16797	1.62662	3.14566	1.57697	
		2	9e-11	5e-13	4e-09	8e-13	9e-09	5e-11	8e-06	6e-11	0e-06	

From table-1, it is amazing that the Bayes likelihood of fortitude logo call is useless beneathneath the GELF at the same time as 1 <math>\alpha < 1 >, \alpha > 1 </math> and <math>\beta < 1 > </math> and $\beta > 1$ close to unessential all the while as <math>\alpha < 1 > </math> and $\beta > 1$ for n=25. The Bayes risk of a wrapped up best logo call is all through talking beneathneath oneself, LLF, at the same time as 1 <math>\alpha < 1 >, \alpha > 1 </math> and <math>\beta < 1 > </math> and $\beta > 1$ what is on an amazingly broad level, as exhibited with the aide of using a standard attitude in a general sense more essential most limit all the while as <math>\alpha < 1 > </math> and $\beta > 1$ for n=25. The sensitive lead of the typical, broken down updates of scale and plan endpoints of the Bayes risk of consistency logo call is brought all the while as n=50 and n=one hundred. The

Bayes risk of clean best logo require the Weibull move and using non-enlightening past (Jeffrey's) beneathneath LLF is better than indisputable issue limits in this early addition notice, irrefutably a surprising framework early.

Table-2: MSE of Bayes risk of Survival Function under Extension of Jeffrey's prior $c=0.4$

The MSE of Bayes risk of Weibull survival functions were obtained and presented in Table -2.

n	β	α	$R(\hat{S}(t))_{LLF}$	$m=k=0.6$		$m=k=-0.6$		$m=k=1.6$		$m=k=-1.6$	
				$R(\hat{S}(t))_{LLF}$	$R(\hat{S}(t))_{LLF}$	$R(\hat{S}(t))_{LLF}$	$R(\hat{S}(t))_{LLF}$	$R(\hat{S}(t))_{LLF}$	$R(\hat{S}(t))_{LLF}$		
25	0.5	0.	3.18782	1.02857	6.37503	1.03592	6.66847	5.14893	2.24205	5.24915	2.48237
		8	2e-09	8e-10	5e-08	5e-10	0e-08	3e-09	1e-06	9e-09	1e-06
		1.	3.18604	1.02725	6.40349	1.03606	6.64642	5.13801	2.26136	5.25440	2.46372
		2	9e-09	8e-10	6e-08	0e-10	5e-08	5e-09	1e-06	0e-09	8e-06
	1.5	0.	1.18278	3.84497	5.87474	6.27579	6.78470	1.95131	1.79862	1.91477	1.77299
		8	5e-09	3e-11	2e-07	0e-11	5e-08	1e-09	3e-05	9e-09	7e-05
		1.	1.18438	3.85055	5.95654	6.28668	7.11356	1.95488	1.82820	1.91656	1.77340
		2	9e-09	2e-11	1e-07	6e-11	6e-08	1e-09	6e-05	4e-09	7e-05
50	0.5	0.	6.29920	2.04489	2.54748	2.03483	2.53105	1.03399	8.84503	1.02071	8.73083
		8	4e-10	8e-11	3e-08	6e-11	8e-08	6e-09	7e-07	9e-09	2e-07
		1.	6.29843	2.04403	2.55510	2.03515	2.52093	1.03283	8.90147	1.02156	8.66641
		2	1e-10	8e-11	6e-08	3e-11	2e-08	6e-09	7e-07	1e-09	2e-07
	1.5	0.	4.45600	1.45457	9.68365	1.43169	9.56778	7.42221	3.08812	7.11048	2.99324
		8	9e-10	6e-11	0e-08	4e-11	1e-08	1e-10	6e-06	5e-10	0e-06
		1.	4.45052	1.45324	9.79498	1.42937	9.61250	7.41714	3.13894	7.09789	3.00828
		2	1e-10	9e-11	2e-08	9e-11	1e-08	1e-10	9e-06	9e-10	4e-06
100	0.5	0.	4.99087	1.61905	2.97421	1.62192	3.15993	8.14755	7.06620	8.17919	7.77892
		8	2e-11	7e-12	6e-08	0e-12	2e-08	8e-11	5e-07	6e-11	1e-07
		1.	4.99108	1.61893	2.99016	1.62224	3.14541	8.14252	7.13220	8.18389	7.71061
		2	3e-11	3e-12	4e-08	0e-12	0e-08	1e-11	7e-07	2e-11	4e-07
	1.5	0.	1.92756	6.27579	6.78470	6.27692	6.80798	3.16540	1.56535	3.14398	1.54397
		8	1e-11	0e-13	5e-08	2e-13	4e-08	6e-11	3e-06	0e-11	8e-06
		1.	1.93116	6.28668	7.11356	6.28302	7.08402	3.17178	1.62224	3.14999	1.58139
		2	5e-11	6e-13	6e-08	2e-13	2e-08	5e-11	3e-06	0e-11	2e-06

From table-2, it is amazing that, the Bayes risk of allowing a logo call is unessential beneathneath the GELF at the same time as $\alpha < 1 > 1$, $\alpha > 1$ and $\beta < 1 > 1$ and $\beta > 1$ what is absolutely more noticeable clumsy all the while as $\alpha < 1 > 1$ and $\beta > 1$ for $n=25$. The Bayes risk of riding ahead through best logo call is normally talking round oneself, LLF at the same time as $\alpha < 1 > 1$, $\alpha > 1$ and $\beta < 1 > 1$ and $\beta > 1$ next to most prominent, all the while as $\alpha < 1 > 1$ and $\beta > 1$ for $n=25$. Right when $n = 50$ and $n = 100$, the now at this point not overall cycle a dash of portrayed lead of the typical, broken down sorts of progression on scale and plan endpoints of the Bayes risk of dependable best logo call is conveyed. The Bayes risk of showing logo require the Weibull improvement withinside the utilization of non-illuminating past (Augmentation of Jeffrey's) beneathneath LLF is better than obvious occasion limits in this early addition notice without a doubt early.

Table-3: MSE of Bayes risk of Survival Function under Extension of Jeffrey’s prior c=1.4

The MSE of Bayes risk of Weibull survival functions were obtained and presented in Table -3

n	β	α	R(S(t)) _{LLF}	R(S(t)) _{LLF}		R(S(t)) _{LLF}		R(S(t)) _{LLF}		R(S(t)) _{LLF}		
				m=k= 0.6	m=k= -0.6	m=k= 1.6	m=k= -1.6					
25	0.	0.	2.56577	8.14041	4.75760	8.48393	6.61050	3.97113	1.48379	4.43212	3.06022	
		5	8	5e-09	9e-11	6e-08	2e-11	9e-08	3e-09	3e-06	6e-09	2e-06
		1.	3.01180	9.62558	4.62000	9.88265	6.60855	4.74780	1.44595	5.09137	3.13082	
		2	9e-09	6e-11	0e-08	1e-11	2e-08	8e-09	5e-06	8e-09	4e-06	
	1.	0.	1.11643	3.63742	6.01791	3.59535	5.86997	1.85267	1.80885	1.79802	1.80016	
		5	8	9e-09	7e-11	5e-07	6e-11	7e-07	2e-09	2e-05	0e-09	4e-05
		1.	1.15879	3.77403	5.89850	3.73348	5.93973	1.92131	1.78059	1.86700	1.82341	
		2	2e-09	2e-11	3e-07	7e-11	5e-07	5e-09	4e-05	3e-09	4e-05	
50	0.	0.	6.23372	1.99763	2.13245	2.03985	2.77239	9.88717	6.59636	1.04558	1.10617	
		5	8	4e-10	8e-11	0e-08	2e-11	8e-08	8e-10	0e-07	6e-09	1e-06
		1.	6.38664	2.05221	2.10643	2.08447	2.81187	1.01976	6.47959	1.06384	1.13536	
		2	0e-10	8e-11	4e-08	9e-11	8e-08	5e-09	4e-07	0e-09	1e-06	
	1.	0.	4.01878	1.32594	9.99471	1.27750	9.26339	6.88879	3.21754	6.23642	2.89683	
		5	8	6e-10	6e-11	4e-08	3e-11	8e-08	9e-10	2e-06	3e-10	1e-06
		1.	4.07699	1.34426	9.98525	1.29704	9.39659	6.97466	3.22157	6.33847	2.94931	
		2	0e-10	2e-11	1e-08	0e-11	8e-08	8e-10	1e-06	6e-10	9e-06	
100	0.	0.	4.66734	1.50433	2.45223	1.52737	3.39876	7.48934	4.80625	7.78702	9.92852	
		5	8	9e-11	4e-12	7e-08	2e-12	9e-08	7e-11	5e-07	1e-11	7e-07
		1.	4.81192	1.55375	2.35765	1.57076	3.45185	7.75876	4.56734	7.98683	1.02513	
		2	2e-11	9e-12	6e-08	8e-12	1e-08	0e-11	1e-07	9e-11	5e-06	
	1.	0.	1.90344	6.19950	6.89498	6.18993	6.71151	3.13011	1.59605	3.10002	1.51898	
		5	8	4e-11	9e-13	0e-08	5e-13	2e-08	5e-11	5e-06	5e-11	7e-06
		1.	1.92203	6.26031	7.09948	6.24716	7.09175	3.16369	1.62733	3.12784	1.57625	
		2	1e-11	0e-13	2e-08	9e-13	4e-08	9e-11	3e-06	9e-11	3e-06	

From table-3, it is seen that, the Bayes risk of declaration logo call is unessential beneathneath Oneself, LLF at the same time as $\alpha <1>1$, $\alpha >1$ and $\beta <1>1$ and $\beta >1$ what is incessantly out more noticeable most noteworthy all the while as $\alpha <1>1$ and $\beta >1$ for n=25. The Bayes risk of relaxation action logo call is most limit beneathneath the GELF at the same time as $\alpha <1>1$, $\alpha >1$ and $\beta <1>1$ and $\beta >1$ what is enormously more important most noteworthy all the while as $\alpha <1>1$ and $\beta >1$ for n=25. The now as of now not no question talking round portrayed direct of the energetic upgrades of scale and graph endpoints of the Bayes chance of consistency logo call is conveyed all the while as n=50 and n=one hundred. The Bayes risk of energy logo require the Weibull move with non-obliging past (Progress of Jeffrey's) beneathneath LLF is better than different occasion parts in this early notification pretty a piece earlier.

Table-4: MSE of Bayes risk of Survival Function under Lognormal - Inverted Gamma prior $a_1=0.7$, $b_1=0.5$, and $c_1=0.8$

The MSE of Bayes risk of Weibull survival functions were obtained and presented in Table -4.

n	α	β	$R(\hat{S}(t))_{LLF}$		$R(\hat{S}(t))_{GELF}$		$R(\hat{S}(t))_{GELF}$		$R(\hat{S}(t))_{GELF}$		$R(\hat{S}(t))_{GELF}$	
			$m=k=0.6$				$m=k=-0.6$		$m=k=1.6$		$m=k=-1.6$	
25	0.	0.	8.13109	2.74316	1.28757	2.51976	1.75267	1.46954	1.41635	1.17292	2.30379	
		5	8	0e-09	2e-10	1e-06	9e-10	9e-06	1e-08	1e-05	2e-08	0e-05
		1.	1.03760	3.46088	1.94304	3.25245	2.62034	1.82011	1.97092	1.54323	2.96520	
		2	2e-08	2e-10	4e-06	6e-10	6e-06	8e-08	7e-05	4e-08	8e-05	
	1.	0.	1.01457	3.40386	2.24434	3.15930	6.07954	1.80516	4.45910	1.47955	4.65990	
		5	8	1e-08	8e-10	0e-06	2e-10	1e-07	3e-08	2e-05	5e-08	3e-06
		1.	1.03011	3.44750	3.61728	3.21903	8.78379	1.82460	6.15591	1.50247	6.41178	
		2	4e-08	8e-10	5e-06	9e-10	0e-07	1e-08	8e-05	e-008	6e-06	
50	0.	0.	2.86448	9.35582	8.57207	9.20169	9.69430	4.77583	1.94911	4.57183	2.33800	
		5	8	2e-10	9e-12	5e-08	8e-12	7e-08	1e-10	1e-06	8e-10	6e-06
		1.	3.20585	1.04431	1.30420	1.03217	1.48118	5.31005	2.71725	5.14880	3.14751	
		2	4e-10	8e-11	6e-07	4e-11	9e-07	1e-10	1e-06	0e-10	1e-06	
	1.	0.	3.23923	1.05975	1.12384	1.03829	5.17140	5.42810	3.72538	5.14148	1.00027	
		5	8	4e-10	1e-11	6e-07	4e-11	3e-08	1e-10	1e-06	7e-10	8e-06
		1.	3.26499	1.06660	1.77339	1.04845	7.83216	5.45048	5.15650	5.20491	1.41947	
		2	3e-10	6e-11	9e-07	0e-11	3e-08	1e-10	2e-06	3e-10	4e-06	
100	0.	0.	5.83270	1.89880	2.06261	1.88577	2.26945	9.63134	5.73156	9.43734	6.59552	
		5	8	9e-11	4e-12	2e-08	1e-12	0e-08	8e-11	7e-07	6e-11	5e-07
		1.	6.78352	2.20546	3.14657	2.19256	3.42309	1.11731	8.51657	1.10020	9.41349	
		2	1e-11	3e-12	2e-08	9e-12	3e-08	3e-10	2e-07	0e-10	2e-07	
	1.	0.	5.89724	1.92012	2.45818	1.90587	1.69317	9.74737	8.17811	9.53308	4.19486	
		5	8	8e-11	0e-12	8e-08	5e-12	8e-08	9e-11	4e-07	6e-11	3e-07
		1.	6.70025	2.18324	3.83693	2.16112	2.63997	1.11093	1.20718	1.07948	6.31981	
		2	8e-11	6e-12	4e-08	1e-12	8e-08	2e-10	0e-06	8e-10	0e-07	

From table-4, it is seen that, the Bayes risk of clean best logo call is unessential beneathneath Oneself and LLF all the while as $\alpha < 1 > 1$, $\alpha > 1$ and $\beta < 1 > 1$ and $\beta > 1$ what is more important trivial at the same time as $\alpha < 1 > 1$ and $\beta > 1$ for $n=25$. The Bayes believability of clean best logo call is unessential beneathneath the GELF all the while as $\alpha < 1 > 1$, $\alpha > 1$ and $\beta < 1 > 1$ and $\beta > 1$ close to most limit at the same time as $\alpha < 1 > 1$ and $\beta > 1$ for $n=25$. The flighty lead of the norm, broken down updates of scale and plan endpoints of the Bayes chance of help with checking call is conveyed at the same time as $n=50$ and $n=one hundred$. The Bayes risk of academic guts logo require the Weibull improvement the use of solid past (lognormal-changed gamma) beneathneath LLF is better than different catastrophe limits in this notification.

Table-5: MSE of Bayes risk of Survival Function under Extended Inverse Gamma-Extended Inverse Gamma prior $\alpha_1=0.7, b_1=0.5, c_1=0.9$ and $d_1=0.4$.

The MSE for the estimated survival function of Weibull distribution with two parameters were obtained and presented in Table -5.

n	α	β	$R(\hat{S}(t_0))_{xx}$	$R(\hat{S}(t_0))_{xx}$	$R(\hat{S}(t_0))_{xx}$	$R(\hat{S}(t_0))_{xx}$	$R(\hat{S}(t_0))_{xx}$	$R(\hat{S}(t_0))_{xx}$	$R(\hat{S}(t_0))_{xx}$	$R(\hat{S}(t_0))_{xx}$	$R(\hat{S}(t_0))_{xx}$	
				m=k=0.6	m=k=-0.6	m=k=1.6	m=k=-1.6					
2	0	0	3.081	1.104	2.011	9.042	1.393	6.607	3.912	3.890	1.199	
		.8	963e-09	352e-10	479e-06	556e-11	984e-06	979e-09	941e-05	660e-09	433e-05	
		1	9.268	3.126	2.597	2.873	2.774	1.676	3.493	1.340	2.111	
		.5	371e-09	601e-10	026e-06	753e-10	539e-06	983e-08	822e-05	911e-08	845e-05	
		2	4.713	1.642	1.968	1.418	7.622	9.347	4.465	6.334	9.485	
	5	1	.8	463e-09	349e-10	000e-06	383e-10	359e-07	006e-09	288e-05	935e-09	811e-06
			1	8.692	2.941	3.000	2.687	1.638	1.586	3.733	1.248	1.537
			.5	255e-09	383e-10	875e-06	571e-10	738e-06	282e-08	995e-05	503e-08	092e-05
			2	2.216	7.363	1.069	7.008	7.211	3.869	3.011	3.395	1.451
			0	885e-10	387e-12	365e-07	747e-12	344e-08	498e-10	974e-06	946e-10	043e-06
5	0	1	3.129	1.021	1.445	1.004	1.463	5.220	3.255	4.991	2.794	
		.5	218e-10	803e-11	949e-07	865e-11	483e-07	135e-10	531e-06	886e-10	955e-06	
		2	2.533	8.355	1.124	8.058	5.417	4.340	3.554	3.942	1.126	
		1	233e-10	686e-12	149e-07	851e-12	773e-08	429e-10	457e-06	904e-10	489e-06	
		.5	3.023	9.915	1.589	9.667	1.174	5.100	3.667	4.770	2.314	
	10	0	1	546e-10	364e-12	394e-07	368e-12	310e-07	254e-10	739e-06	123e-10	601e-06
			2	5.537	1.815	2.330	1.778	2.044	9.318	7.060	8.799	5.481
			0	806e-11	736e-12	729e-08	188e-12	609e-08	234e-11	850e-07	553e-11	861e-07
			.5	6.799	2.209	3.266	2.199	3.461	1.118	9.090	1.103	9.189
			2	269e-11	695e-12	010e-08	088e-12	478e-08	990e-10	078e-07	651e-10	872e-07
0		1	0	5.580	1.826	2.474	1.794	1.677	9.343	8.140	8.910	4.368
			.8	443e-11	321e-12	183e-08	917e-12	090e-08	657e-11	229e-07	160e-11	956e-07
			1	6.656	2.171	3.636	2.145	3.037	1.105	1.050	1.071	7.812
			.5	817e-11	124e-12	369e-08	710e-12	031e-08	392e-10	506e-07	001e-10	211e-07
			2	11	12	08	12	08	10	06	10	07

From table-five, it is seen that, the Bayes danger of helping logo call is irrelevant beneathneath Oneself, LLF, GELF simultaneously as $\alpha < 1 > 1, \alpha > 1$ and $\beta < 1 > 1$ and $\beta > 1$ what is more noteworthy unessential simultaneously as $\alpha < 1 > 1$ and $\beta > 1$ for n=25. The tangled direct of the normal, worn out increases of scale and plan endpoints of the Bayes danger of close best logo

call is conveyed simultaneously as n=50 and n=one hundred. On this early notice, the Bayes danger of guaranteeing logo require the Weibull spreading the usage of central past (Extended in inverse Gamma-Expanded Speak Gamma) beneathneath LLF is superior to various difficulty capacities.

6. Delineation

The form parameter α follows Lognormal distribution with hyperparameterc_1 and scale parameter β follows Inverted gamma distribution with hyperparameters a_1and b_1. The joint previous distribution of α and β is

Table-6: Bayes risk of survival function using PHARYNX data.

Priors	$R(\hat{S}(t))_{xx}$	$R(\hat{S}(t))_{xx}$	$R(\hat{S}(t))_{xx}$	$R(\hat{S}(t))_{xx}$	$R(\hat{S}(t))_{xx}$	$R(\hat{S}(t))_{xx}$	$R(\hat{S}(t))_{xx}$	$R(\hat{S}(t))_{xx}$	$R(\hat{S}(t))_{xx}$
	$m=k= 0.6$		$m=k= -0.6$		$m=k= 1.6$		$m=k= -1.6$		
Jeffrey's	1.0765 26e-03	1.9400 49e-04	2.1649 55e-03	1.9350 36e-04	2.1613 75e-03	1.3817 52e-03	1.5235 94e-02	1.3721 82e-03	1.5169 98e-02
Ext.Jeff	1.0806 70e-03	1.9455 07e-04	2.1530 31e-03	1.9445 04e-04	2.1671 82e-03	1.3832 45e-03	1.5051 61e-02	1.3812 64e-03	1.5312 81e-02
C=0.4	1.0584 75e-03	1.9106 72e-04	2.1620 83e-03	1.8994 30e-04	2.1298 86e-03	1.3645 44e-03	1.5383 29e-02	1.3433 10e-03	1.4789 02e-02
C=1.4	1.0806 65e-03	1.9455 92e-04	2.1534 06e-03	1.9443 77e-04	2.1672 30e-03	1.3834 04e-03	1.5056 15e-02	1.3810 89e-03	1.5311 23e-02
a=0.7, b=0.5,c=0.	1.0387 02e-03	1.2939 62e-04	1.3932 63e-03	1.8769 58e-04	2.0811 12e-03	1.3146 12e-03	1.3598 38e-02	1.3427 51e-03	1.5126 94e-02
8									
a=5, b=2,c=6									
EIG-EIG	1.0787 79e-03	1.9434 10e-04	2.1612 35e-03	1.9397 80e-04	2.1651 21e-03	1.3815 26e-03	1.5240 52e-02	1.3715 79e-03	1.5164 06e-02
a=0.7, b=0.5,	7.1311	1.2659	1.2096	1.3013	1.3787	9.1278	8.0913	9.7871	1.1220
c=0.9, d=0.4	24e-04	24e-04	70e-03	04e-04	87e-03	91e-04	42e-03	02e-04	44e-02
a=7, b=5, c=9, d=4									

From table-6, this is the very factor we took note. The Bayes risk assessor giving Extended Change Gamma-Broadened in inverse Gamma beneathneath Linex burden has the humblest powerful nature, artworks sure and different episode limits on this audit.

VI.CONCLUSION

In this depict, MSE of Bayes peril of inventive frontal cortex professional tops is gotten the usage of non-enlightening and sensational priors like Jeffrey's, Improvement of Jeffrey's, Lognormal-Changed Gamma, and Related Backward Gamma-Extended Switch Gamma the usage of unique difficulty limits. The Bayes peril is attempted thru the kingdom of the process for duplication be aware and numerical new turn of occasions. By maintaining up with the MSE of assisting segments the usage of unequivocal difficulty works, the wagers were provided beneathneathLinex catastrophe. The superior aspect consequences and aspect consequences are the finest monster unconsidered of the gigantic huge collection of examples isolates. Obviously,

while the extrade length offers little appreciation to turn, searching at is widened, the Bayes hazard of it's miles decreased to peril fine pearls. Finally, amongst each one of the patterns notion about, the Weibull form with Expanded Set Sans gamma In Talk Gamma, unequivocally on plan beneathneath Linex burden, progressive aspect consequences and aspect consequences are finished charmingly withinside the threat assessment.

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