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Generalized Fuzzy Entropy Measures and Their Properties

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ABSTRACT
The fuzzy theory is a practical mathematical device to contract with the uncertainties. Latterly, the uncertainty measures of fuzzy sets have attained considerations from the researchers. This paper is concerned on the subject of the different uncertainty measures of fuzzy sets. Several theorems and properties of each fuzzy entropy measure are presented. It also represents the latest theories which introduce the different measures like similarity measure, distance measure, divergence measure and interval entropy of fuzzy sets. The applications of the fuzzy entropy measures and their relationships among each other from the literatures are also studied.

Keyword: Fuzzy Set Theory, Fuzzy Sets, Fuzzy Entropy Measures, Divergence Measures.

I. INTRODUCTION

A fundamental question regarding the assumptions surrounding vulnerabilities is Vulnerability Measure. The idea of a fluffy set suggests that the vulnerability can be easily controlled in some way. For really looking at the fluffiness, the framework of the entropy hypothesis is the best in class for estimating the vulnerability of a likelihood dissemination. The concept of fluffy sets has emerged as an extreme area of investigation in various methods. The numerical standards for actually examining the fluffiness of fluffy sets are referred to as fluffy entropy. As the primary subject of the data hypothesis, entropy should be used to really look at how fluffy the fluffy sets are.

Entropy, first proposed by Shannon [1], was the essential proportion of data. A fundamental hypothesis for estimating the fluffy information was fluffy entropy. In 1965, Zadeh [2] looked at the Fluffy sets hypothesis, which had been developed into a widespread machine that was used in numerous fields of science and technology. Fluffiness was a vulnerability that developed in response to a lack of fine difference between one or more members of the group. In 1968, Zadeh developed an initial plan to register the fluffy entropy [3], and a likely conspiracy also portrayed the fluffy entropy's reality as the standard Shannon entropy [1]. De Luca [4] used fluffy entropy gauges to represent hypotheses and looked at the entropy of a fluffy set based on Shannon's capability. Similar to Kullback-Leibler's [6,7] measure, Bhandari [5] recommended a fluffy dissimilarity measure between two fluffy sets. Shang [9] later imported the fluffy dissimilarity measure that was suggested in [5] to adopt the concept of Lin [8]. In the hypothesis of data for the end of entropy of Shannon's entropy, Karridge [10] largely introduced the concept of proportional error in 1961. He said that the proportion of errors as a percentage of the limit of lost data was the one that was more dangerous. Verma and Sharma [11] determined an error measure for the methodology of fluffy sets in light of a questionable encounter with wrapped non-probabilistic structures. Kapur [12] questioned the vulnerability proportions of fluffy entropy from the point of view of fluffiness, but not from the point of view of probabilistic entropy from the point of view of information, which is material to the state of a dissemination of likelihood, as it were. A fluffy entropy measure based on the measurement distance between the fluffy sets was imported by Kaufmann [13]. As a kind of "measure," Yager [14] depicted the entropy of fluffy without any distinction between its arrangement and its reverse. Parkash et al. in 2008 [15] presented two brand-new estimates of fluffy entropy based on geometrical capabilities and a checked entropy expansion premise that were comparable to these fluffy entropies. In addition, a variety of research projects were carried out on the applications of these academic studies [16, 17, 18, and 19]. For estimating the fluffiness' certification, Hung and Yang [20] imported two proportions of entropy and introduced the more stable suggested measure. Another entropy measure, the end of the Shannon entropy, was proposed by Dish and Deng [21] for estimating the likelihood conveyance's vulnerability.

Kullback and Leibler [22] introduced the difference's action right away, which was a measure of how different the reproduced likelihood circulation was from the real dispersion. Another fluffy measure for the bias of one fluffy set in relation to the next fluffy set was provided by Buddy and Bhandari [23]. Fan and Xie [24] imported the proportion of fluffy difference that was outlined on the activity of remarkable and looked at the relationship with the proportion of fluffy disparity that was imported in [23]. Montes and others 25] contemplated the specific order of disparity proportions and their connection to the concern of the fluffy hypothesis. Parkash and others 26] proposed two distinct measures to contrast Ferreri's [27] coordinated uniqueness measure's proportions. To consider Taneja's [29] proportion of number-crunching mathematical disparity, Singh and Bhatia [28] recommended the fluffy difference measure. Jain and Hooda [30] acknowledged that there was a noticeable way to pinpoint the growth of data and vulnerability. Tomar and Ohlan [31, 32] took into account a variety of proportions of fluffy mean dissimilarity by distinguishing between them and incorporating their apparatuses in the states of example recognition and the adaptability in phonetic factors. To examine Verma and Sharma's [34] measures, Tomar [33] imported a brand-new parametric summarised outstanding fluffy dissimilarity measure.

The proportions of fluffy entropy were the fundamental subjects that a few scientists had thought about frequently in the fluffy set hypothesis based on distant sentiments. Examining the "similitude measures," Wang [35] first proposed a percentage of similarity between two fluffy sets. In order to advance data, Salton and McGill [36] imported a portion of cosine comparability between the arrangements of fluffy, which in premise is a kind of "coefficient." Zwick and others 37] demonstrated the closeness measures by employing mathematical region and Huasdorff measurements between the fluffy sets. Karacapilidis and Pappis [38] proposed the three different similarity measures for the soft sets spread out on the different exercises. Chen and others 39] continued with crafted by Pappis and Karacapilidis [38], and portrayed two or three closeness feathery measures. Wang [40] proposed two proportions of comparability between individuals from the sets and the fluffy sets. A specific assurance of the degree of proximity was brought by Liu [41] in addition to Fan and Xie [42]. In addition, a collection of proportions of similarity that Szimidt and Kacprzyk [43] did not completely define joined it by a few real proportions of closeness. Kim and Hong [44], Yang and Hung [45] and Xu [46] portrayed the further extents of likeness by applying specific distance measures. Similarly, Hung and Yang affirmed several assets of a general measure and related it by the general measures and the relationship showed its general extent of likeness which was adequate smooth that of the general measures. Taking into account a number of new likeness measures, Garg and Kumar [47] explained the gap in a few general estimates. The proportions of entropy can be used to lay out the proportions of comparability, as suggested by Wei and Wang [48]. A proceeded approach for framework decisions based on a Pythagorean fluffy set was proposed by Zhang and Xu [49]. Using the entropy hypotheses, sufficient writing about the vulnerability was introduced (see [50-57]. Jin and co. 56] exhorted the fluffy persistent weighted entropy depicted in Szmidt and Kacprzyk's actions to continue. In addition to the aforementioned findings, a few papers focused on the relationships between entropy and various measures; see [58-65].

Its broad applications in various fields were comprehensively viewed as the proportion of likeness, the distance measure, entropy, and its connections [66-78]. The connections between the various measures were taken into consideration. These investigations provided a fundamental, though potentially intriguing, and research direction. On the other hand, a few fundamental issues actually call for additional investigation. The extents of closeness introduced in [79] were genuinely limited so much that it took only the courses of action of feathery by the essential game plan of limit thoroughly. The proportions of likeness and distance in [80-82] revealed a comparable issue. It was expected to determine the various proportions of likeness and entropy in order to bargain with the unmistakable potential issues and provided a broad connection to previously mentioned ideas. A philosophy of cushioned sets introduced by Zadeh had been found strong in the conditions of the different fields. The significant points that had been examined by the various analysts associated with the fluffy setting included measures of entropy, distance, and comparability. The similarity or dissimilarity of the two fluffy sets was determined by these actions. Fleecy sets achieved a fundamental idea from the investigates as its applications in the different fields.

This paper considers the different extents of weakness like difference measures, comparability measures, stretch measures, distance measures and cosine measures for the fleecy sets. It is divided into four sections: Segment I covers the presentation, and Segment II looks at some ideas, concepts, definitions, and properties associated with fluffy sets. The same investigation of the various vulnerability measures and entropy for fluffy sets that are mentioned in the written works is presented in Segment III. Also, relations between the different measures like range closeness, stretch distance and stretch consolidation measures have been moreover researched. The uses of the various fluffy entropy measures are shown in Section IV, and the conclusion of the paper is detailed in Section V

II. PRELIMINARIES

The following section represents the concepts of the fuzzy set theory that will require in the following study:

Let Z be any set. Then, the mapping $\varphi_F: Z\rightarrow[0,1]$ is known as a fuzzy subset of Z. Let $FS(Z)$and $CS(Z)$ be the family of all the fuzzy sets and non-fuzzy sets over Z, respectively.

Fuzzy Set (FS) [2]: A set of fuzzy F described in a definite universe of discourse $Z=\{z_1,z_2,\ldots,z_m\}$ is mathematical represented as:

$$F=\{\langle z,\varphi_F(z)\rangle \mid z\in Z\}, \quad (1)$$

For $\varphi_F(z):Z\rightarrow[0,1]$ be the function of membership of F.

The value of $\varphi_F(z)$ defines the degree of the belongingness of an element $z\in Z$ in F.If $\varphi_F(z)$ is admired in $\{0,1\}$, then it becomes the specific or essential function of a non-fuzzy set.

If the following conditions are fulfilled then any fuzzy set F^* is known as a sharpened form of fuzzy set F: $\varphi_{F^*}(z_j)\leq\varphi_F(z_j)$ if $\varphi_F(z)\leq 0.5\,\forall j$,

$\varphi_{F^*}(z_j)\geq\varphi_F(z_j)$ if $\varphi_F(z)\geq 0.5\,\forall j$,

where $j=1,2,\ldots,m$.

Fuzzy Sets Operations [83]: If $FS(Z)$ indicates the group of whole the fuzzy sets in the universe Z, and $F_1,F_2\in FS(Z)$ be two FSs, given by,

$$F_1=\{\langle z,\varphi_{F_1}(z)\rangle \mid z\in Z\},$$

$$F_2=\{\langle z,\varphi_{F_2}(z)\rangle \mid z\in Z\},$$

Then, the following set operations are defined as:

(i) $F_1\subseteq F_2$ if and only if $\varphi_{F_1}(z)\leq\varphi_{F_2}(z)\,\forall z\in Z$;

(ii) $F_1=F_2$ if and only if $F_1\subseteq F_2$ and $F_2\subseteq F_1$;

(iii) $F_1^C=\{\langle z,1-\varphi_{F_1}(z)\rangle \mid z\in Z\}$;

(iv) $F_1\cap F_2=\{\langle z,\varphi_{F_1}(z)\vee\varphi_{F_2}(z)\rangle \mid z\in Z\}$;

(v) $F_1\cup F_2=\{\langle z,\varphi_{F_1}(z)\wedge\varphi_{F_2}(z)\rangle \mid z\in Z\}$;

where $\vee$, $\wedge$ denote the maximum and minimum operators, respectively.

[64] A number which described for the function of membership$\varphi_F(z)$ as a fuzzy subset is known as a fuzzy number, for any $z\in R$ which fulfilles the consecutive assets:

(i) If F be a normal fuzzy set; i.e.,∃ z∈R,φ_F (z)=1.

(ii)If F be a convex fuzzy set; i.e., for any z_1,z_2 and δ∈[0,1],

φ_F (δz_1+(1-δ)z_2)≥min(φ_F (z_1),φ_F (z_2)).

(iii) The support of F is bounded; i.e., the set Supp(F)={z∈R|φ_F (z)>0} is bounded.

III MEASURES OF FUZZY ENTROPY

This section deals with the following different fuzzy entropy measures:

(i) Fuzzy Entropy Measures:
[84]Fuzziness is a component of uncertainty which develops in distinction to the absence of sharpened difference of either a member of set or not a member of the set.
Fuzzy entropy measure is a measure of the fuzziness of the fuzzy set.
'Entropy' was firstly used by Shannon [1] in probability distribution for measuring a degree of the randomness and described the instructions which involved in that search. It is inclined as:

$$E(D) = -\sum_{j=1}^{m} d_j \log d_j \qquad (2)$$

where $D = (d_1, d_2, \ldots, d_m)$ be the probability distributions in an experiment.
Termini and De Luca [85] imported the fuzzy entropy measure for comparing Shannon's entropy which is inclined in (2) as:

$$E_1(F) = -\sum_{j=1}^{m}\left[\varphi_F(z_j)\log\varphi_F(z_j) + \left(1-\varphi_F(z_j)\right)\log\left(1-\varphi_F(z_j)\right)\right] \qquad (3)$$

Pal and Pal [86] imported the exponential fuzzy entropy for F based on exponential function as:

$$e^{E_1(F)} = \frac{1}{m(\sqrt{e}-1)}\sum_{j=1}^{m}\left[\varphi_F(z_j)e^{1-\varphi_F(z_j)} + \left(1-\varphi_F(z_j)\right)e^{\varphi_F(z_j)} - 1\right] \qquad (4)$$

Later on, Pal and Bhandari [83] built an analysis for the measures of entropy of the sets of fuzzy also imported the parametric fuzzy entropy for the set of fuzzy F just as:

$$E_\gamma(F) = \frac{1}{n(1-\gamma)}\sum_{j=1}^{m}\log\left[\varphi_F^{\gamma}(z_j) + \left(1-\varphi_F(z_j)\right)^{\gamma}\right] \qquad (5)$$

Parkash et al. [15] defined two fuzzy entropy measures for fuzzy set F based on trigonometric functions as:

$$E_{sin}(F) = \frac{1}{m}\sum_{j=1}^{m}\left[\left\{\sin\frac{\pi\varphi_F(z_j)}{2} + \sin\frac{\pi\left(1-\varphi_F(z_j)\right)}{2} - 1\right\}\frac{1}{(\sqrt{2}-1)}\right] \qquad (6)$$

$$E_{cos}(F) = \frac{1}{m}\sum_{j=1}^{m}\left[\left\{cos\frac{\pi\varphi_F(z_j)}{2} + cos\frac{\pi(1-\varphi_F(z_j))}{2} - 1\right\}\frac{1}{(\sqrt{2}-1)}\right] \quad (7)$$

After these fuzzy entropy measures, a lot of developments and conclusions showed in the research.

(ii) Measures of Fuzzy Divergence:

Leibler and Kullback [22] attained the directed divergence measure of the distribution of probability $P = (p_1, p_2, \ldots, p_m)$ by probability distribution $Q = (q_1, q_2, \ldots, q_m)$ as:

$$E(P|Q) = \sum_{j=1}^{m} p_j \log\frac{p_j}{q_j} \quad (8)$$

Pal and Bhandari [23] imported fuzzy directed divergence measure related to (5) just as:

$$E(F_1, F_2) = \sum_{j=1}^{m}\left[\varphi_{F_1}(z_j)\log\frac{\varphi_{F_1}(z_j)}{\varphi_{F_2}(z_j)} + \left(1 - \varphi_{F_1}(z_j)\right)\log\frac{1-\varphi_{F_1}(z_j)}{1-\varphi_{F_2}(z_j)}\right] \quad (9)$$

satisfying the conditions:

$(i) E(F_1, F_2) \geq 0,$

$(ii) E(F_1, F_2) = 0 \text{ if and only if } F_1 = F_2,$

$(ii) E(F_1, F_2) \text{ is a convex function of } \varphi_F(z_j).$

Next, Shang and Jiang [9] indicated the failing of (9) that if $\varphi_{F_2}(z_j)$ tends to 0 or 1 then the value of $\varphi_{F_2}(z_j)$ moves to ∞ and recommended a new form of fuzzy measure of (9) which inclined as:

$$J(F_1, F_2) = \sum_{j=1}^{m}\left[\varphi_{F_1}(z_j)\log\frac{\varphi_{F_1}(z_j)}{\frac{\varphi_{F_1}(z_j)+\varphi_{F_2}(z_j)}{2}} + \left(1 - \varphi_{F_1}(z_j)\right)\log\frac{1-\varphi_{F_1}(z_j)}{1-\left(\frac{\varphi_{F_1}(z_j)+\varphi_{F_2}(z_j)}{2}\right)}\right] \quad (10)$$

Couso et al.[87] described the new measure which is as below:

Let Z be a universe of discourse and FS (Z) be the set of the group of subsets of a fuzzy set, then a mapping $M: FS(Z) \times FS(Z) \rightarrow R$ be a fuzzy divergence measure which is given for each $F_1, F_2, F_3 \in FS(Z)$, if the following axioms hold:

$(i) M(F_1, F_2) = M(F_2, F_1),$

$(ii) M(F_1, F_1) = 0,$

$(iii) \max\{M(F_1 \cup F_3, F_2 \cup F_3), M(F_1 \cap F_3, F_2 \cap F_3)\} \leq M(F_1, F_1).$

Non-negativity of $M(F_1,F_2)$ is the natural assumption.

The parametric generalized fuzzy divergence measure between two fuzzy sets F_1,F_2 having membership value $\in (0,1)$ corresponding to Taneja [88] new generalized triangular discrimination measure. It is given by

$$D_f(F_1,F_2)=\sum_{j=1}^{m}\frac{\left(\varphi_{F_1}(z_j)-\varphi_{F_2}(z_j)\right)^2}{2^f}$$
$$\times\left[\frac{\left(\varphi_{F_1}(z_j)-\varphi_{F_2}(z_j)\right)^f}{\left(\sqrt{\varphi_{F_1}(z_j)\varphi_{F_2}(z_j)}\right)^{f+1}}+\frac{\left(2-\varphi_{F_1}(z_j)-\varphi_{F_2}(z_j)\right)^f}{\left(\sqrt{\left(1-\varphi_{F_1}(z_j)\right)\left(1-\varphi_{F_2}(z_j)\right)}\right)^{f+1}}\right],f$$
$$=0,1,2,\ldots \qquad (11)$$

Note: $D_f(F_1,F_2)$ is an accurate measure.

Different Claims of Fuzzy Divergence Measures:

$(i)D_f(F_1\cup F_2,F_1\cap F_2)=D_f(F_1,F_2).$

$(ii)D_f(F_1\cup F_2,F_1)+D_f(F_1\cap F_2,F_1)=D_f(F_1,F_2).$

$(iii)D_f(F_1\cup F_2,F_3)\leq D_f(F_1,F_3)+D_f(F_2,F_3).$

$(iv)D_f(F_1\cup F_2,F_3)+D_f(F_1\cap F_2,F_3)=D_f(F_1,F_3)+D_f(F_2,F_3).$

$(v)D_f(F_1,F_1\cap F_2)=D_f(F_2,F_1\cup F_2).$

$(vi)D_f(F_1,F_1\cup F_2)=D_f(F_2,F_1\cap F_2).$

$(vii)D_f(F_1,F_1)=D_f(F_1,F_1).$

$(viii)D_f(F_1,F_2)=D_f(F_1,F_2).$

$(ix)D_f(F_1,F_2)=D_f(F_1,F_2).$

$(x)D_f(F_1,F_2)+D_f(F_1,F_2)=D_f(F_1,F_2)+D_f(F_1,F_2).$

(iii) Inaccuracy Fuzzy Measures:

[89]Let $F_1\, and F_2$be any two fuzzy sets described in a definite universe of discourse $Z=\{z_1,z_2,\ldots,z_m\}$ having the membership values $\varphi_{F_1}(z_j)\; and\; \varphi_{F_2}(z_j), j=1,2,\ldots,m$ respectively.

A measure of inaccuracy of a fuzzy set F_2 corresponding to a fuzzy set F_1 is defined as:

$$I_1(F_1,F_2)=$$
$$-\frac{1}{m}\sum_{j=1}^{m}\left[\begin{array}{c}\varphi_{F_1}(z_j)log\left(\frac{\varphi_{F_1}(z_j)+\varphi_{F_2}(z_j)}{2}\right)\\+\left(1-\varphi_{F_1}(z_j)\right)log\left(\frac{2-\varphi_{F_1}(z_j)-\varphi_{F_2}(z_j)}{2}\right)\end{array}\right]$$

(12)

$where\; I_1(F_1,F_2)$ is a definite measure which is free from the values of $\varphi_{F_1}(z_j)\; and\; \varphi_{F_2}(z_j)$.

Verma and Sharma [11] later on described an inaccuracy measure of a fuzzy set F_2 corresponding to a fuzzy set F_1, to compare Karridge [10] measure of inaccuracy which is inclined as:

$$I_2(F_1,F_2) = -\frac{1}{m}\sum_{j=1}^{m}\left[\varphi_{F_1}(z_j)log\varphi_{F_2}(z_j) + \left(1-\varphi_{F_1}(z_j)\right)log\left(1-\varphi_{F_2}(z_j)\right)\right] \quad (13)$$

From (12) and (13), it is concluded that $I_1(F_1,F_2)$ can be described in the terms of $I_2(F_1,F_2)$ as:

$$I_1(F_1,F_2) = I_2\left(F_1,\frac{F_1+F_2}{2}\right) \quad (14)$$

It is also noticed that if $F_1 = F_2$, then the measure (12) diminishes to (3).

Properties of Inaccuracy Fuzzy Measure:

$(i) For\ F_1,F_2 \in FS(Z), I_1(F_1,F_2) \le \frac{1}{2}[I_2(F_1,F_2) + E_1(F_1)].$

$(ii) I_1(F_1,F_2) = 0\ iff\ either\ \varphi_{F_1}(z_j) = \varphi_{F_2}(z_j) = 0\ or\ \varphi_{F_1}(z_j) = \varphi_{F_2}(z_j) = 1$
$\forall j = 1,2,\dots,m.$

$(iii)\ For\ F_1,F_2,F_3 \in FS(Z), I_1(F_1 \cup F_2,F_3) \le I_1(F_1,F_3) + I_1(F_2,F_3);$
$I_1(F_1 \cap F_2,F_3) \le I_1(F_1,F_3) + I_1(F_2,F_3).$

$(iv)\ For\ F_1,F_2,F_3 \in FS(Z), I_1(F_1 \cup F_2,F_3) + I_1(F_1 \cap F_2,F_3) = I_1(F_1,F_2) + I_1(F_1,F_3).$

$(v) For\ F_1,F_2 \in FS(Z), I_1(F_1,F_1 \cup F_2) + I_1(F_1,F_1 \cap F_2) = I_1(F_1,F_2) + E_1(F_1);$
$I_1(F_2,F_1 \cup F_2) + I_1(F_2,F_1 \cap F_2) = I_1(F_2,F_1) + E_1(F_2).$

$(vi) For\ F_1,F_2 \in FS(Z),$

$$I_1(F_1 \cup F_2,F_1 \cap F_2) + I_1(F_1 \cap F_2,F_1 \cup F_2) = I_1(F_1,F_2) + I_1(F_2,F_1) = 2E_1\left(\frac{F_1+F_2}{2}\right).$$

$(vii)\ For\ F_1,F_2,F_3 \in FS(Z),$

$$I_1(F_1,F_2 \cup F_3) + I_1(F_1,F_2 \cap F_3) = I_1(F_1 \cup F_2,F_3) + I_1(F_1 \cap F_2,F_3) = I_1(F_1,F_2) + I_1(F_1,F_3).$$

$(viii) For\ F_1,F_2 \in FS(Z), E_1(F_1) \le I_1(F_1,F_2), iff\ F_1 = F_2, i.e., \varphi_{F_1}(z_j) = \varphi_{F_2}(z_j) \forall z_j \in Z.$

Let $F_1\ and F_2$ be any two fuzzy sets described in a definite universe of discourse $Z = \{z_1,z_2,\dots,z_m\}$ having the membership values $\varphi_{F_1}(z_j)\ and\ \varphi_{F_2}(z_j), j = 1,2,\dots,m$, with weight vector $\omega = (\omega_1,\omega_2,\dots,\omega_m)^T$ of the elements of $z_j, where\ j = 1,2,\dots,m$, in case if $\omega_j \ge 0$ and $\sum_{j=1}^{m}\omega_j = 1$, then the weighted fuzzy inaccuracy measure of a fuzzy set F_2 corresponding to a fuzzy set F_1, which inclined as:

$$I_1(F_1,F_2;\omega) = -\sum_{j=1}^{m}\omega_j\left[\varphi_{F_1}(z_j)log\left(\frac{\varphi_{F_1}(z_j)+\varphi_{F_2}(z_j)}{2}\right) + \left(1-\varphi_{F_1}(z_j)\right)log\left(\frac{2-\varphi_{F_1}(z_j)-\varphi_{F_2}(z_j)}{2}\right)\right] \quad (15)$$

If $\omega = \left(\frac{1}{m}, \frac{1}{m}, \dots, \frac{1}{m}\right)$, then (15) is reduced to (12).

(iv) Similarity Fuzzy Measures:
[90]A real function $SM: FS(Z) \times FS(Z) \rightarrow [0,1]$ is known as a similarity fuzzy measure, when SM fullfiles the consecutive claims:
$(i) 0 \leq SM(F_1, F_2) \leq 1 \; \forall \; F_1, F_2 \in FS(Z).$
$(ii) SM(F_1, F_2) = SM(F_2, F_1) \; \forall \; F_1, F_2 \in FS(Z).$
$(iii) SM(F_1, F_2) = 1 \; iff \; F_1 = F_2, i.e. \; \varphi_{F_1}(z_j) = \varphi_{F_2}(z_j) \forall j = 1,2, \dots, m.$
$(iv) For \; F_1, F_2, F_3 \in FS(Z), if \; F_1 \subseteq F_2 \subseteq F_3, then$
$SM(F_1, F_3) \leq SM(F_1, F_2) \; and \; SM(F_1, F_3) \leq SM(F_2, F_3).$

(v) Cosine Fuzzy Measures:
[90]If a fuzzy set F is described in a definite universe of discourse $Z = \{z_1, z_2, \dots, z_m\}$ having $\varphi_{F_1}(z_j)$ as the membership values where $j = 1,2, \dots, m.$

Then, cosine fuzzy entropy for F is defined just as:

$$E_{cos}(F) = \frac{1}{m} \sum_{j=1}^{m} \left[cos\left(\frac{2\varphi_F(z_j) - 1}{2} \pi \right) \right] \tag{16}$$

Propositions of Cosine Fuzzy Measure:
$(i) Sharpness: E_{cos}(F) \; is \; minimum \; iff \; F \; is \; a \; crisp \; set, i.e. \varphi_F(z_j) = 0 \; or \; 1 \; \forall \; z_j \in Z.$
$(ii) Maximality: E_{cos}(F) \; is \; maximum \; iff \; F \; is \; a \; most \; fuzzy \; set, i.e. \varphi_F(z_j) = 0.5 \; \forall z_j \in Z.$
$(iii) Resolution: E_{cos}(F^*) \leq E_{cos}(F), where \; F^* \; is \; a \; sharpened \; version \; of \; the fuzzy \; set \; F.$
$(iv) Symmetry: \; E_{cos}(F) = E_{cos}(F^C), where \; F^C \; is \; the \; complement \; set \; of \; the \; fuzzy \; set \; F$
.
Properties of Cosine Fuzzy Measures:
$(i) If \; F_1, F_2 \in FS(Z) be \; given \; by$
$F_1 = \{\langle z, \varphi_{F_1}(z) \rangle | z \in Z\} \; and \; F_2 = \{\langle z, \varphi_{F_2}(z) \rangle | z \in Z\}$
$such \; that \; they \; satisfy \; for \; any \; z_j \; either \; F_1 \subseteq F_2 or F_1 \supset F_2, then$
$E_{cos}(F_1 \cup F_2) + E_{cos}(F_1 \cap F_2) = E_{cos}(F_1) + E_{cos}(F_2).$
$(ii) If \; F \in FS(Z) \; and \; F^C \; is \; the \; complement \; set \; of \; the \; fuzzy \; set \; F, then$
$E_{cos}(F) = E_{cos}(F^C) = E_{cos}(F \cup F^C) = E_{cos}(F \cap F^C).$

(vi) Cosine Fuzzy Similarity Measures:

Let F_1 and F_2 be any two fuzzy sets described in a definite universe of discourse $Z = \{z_1, z_2, \ldots, z_m\}$ having the membership values $\varphi_{F_1}(z_j)$ and $\varphi_{F_2}(z_j), j = 1,2, \ldots, m$ respectively.

Then, cosine fuzzy similarity measure between the fuzzy sets F_1 and F_2 is defined as:

$$SM_{cos}(F_1, F_2) = \frac{1}{m}\sum_{j=1}^{m}\left[cos\left(\frac{\varphi_{F_1}(z_j) - \varphi_{F_2}(z_j)}{2}\pi\right)\right] \quad (17)$$

Properties of Cosine Fuzzy Similarity Measures:

$(i) 0 \le SM_{cos}(F_1, F_2) \le 1 \; \forall \; F_1, F_2 \in FS(Z).$

$(ii) SM_{cos}(F_1, F_2) = SM_{cos}(F_2, F_1) \; \forall \; F_1, F_2 \in FS(Z).$

$(iii) SM_{cos}(F_1, F_2) = 1 \; iff \; F_1 = F_2, i.e. \varphi_{F_1}(z_j) = \varphi_{F_2}(z_j) \; \forall \; j = 1,2, \ldots, m.$

$(iv) For \; F_1, F_2, F_3 \in FS(Z), if \; F_1 \subseteq F_2 \subseteq F_3, then$
$SM_{cos}(F_1, F_3) \le SM_{cos}(F_1, F_2) \; and \; SM_{cos}(F_1, F_3) \le SM_{cos}(F_2, F_3).$

$(v) For \; F_1, F_2 \in FS(Z), if \; they \; satisfy \; that \; for \; any \; z_j \in Z, either \; F_1 \subseteq F_2 or \; F_1 \supset F_2, then$
$SM_{cos}(F_1 \cup F_2, F_1 \cap F_2) = SM_{cos}(F_1, F_2).$

$(vi) For \; F_1, F_2, F_3 \in FS(Z), then \; SM_{cos}(F_1 \cup F_2, F_3) \le SM_{cos}(F_1, F_3) + SM_{cos}(F_2, F_3) and$
$SM_{cos}(F_1 \cap F_2, F_3) \le SM_{cos}(F_1, F_3) + SM_{cos}(F_2, F_3).$

$(vii) For \; F_1, F_2, F_3 \in FS(Z), then$
$SM_{cos}(F_1 \cup F_2, F_3) + SM_{cos}(F_1 \cap F_2, F_3) \le SM_{cos}(F_1, F_3) + SM_{cos}(F_2, F_3).$

$(viii) For \; F_1, F_2 \in FS(Z), then \; SM_{cos}(F_1, F_2) = SM_{cos}(F_1^C, F_2^C); SM_{cos}(F_1^C, F_2) = SM_{cos}(F_1, F_2^C); SM_{cos}(F_1, F_2) + SM_{cos}(F_1^C, F_2) = SM_{cos}(F_1^C, F_2^C) + SM_{cos}(F_1, F_2^C).$

$(ix) For \; each \; F \in FS(Z),$
$SM_{cos}(F, F^C) = E_{cos}(F), where \; F^C \; is \; the \; complement \; set \; of \; the \; fuzzy \; set \; F.$

(vii) Interval Entropy of Fuzzy Sets:

$The \; real \; function \; I_f : FS(Z) \longrightarrow R([0,1]) F \longmapsto I_f(F) is \; called \; interval \; entropy \; on \; FS(Z),$
$if \; I_f \; has \; the \; following \; propositions:$

$(i) If \; F \in CS(Z), then \; I_f(F) = [0,0].$

$(ii) If \; \forall \; z \in Z, \varphi_F(z) \equiv \frac{1}{2}, then \; I_f(F) = [1,1].$

$(iii) If \; \forall \; z \in Z, either \; \varphi_{F_2}(z) \le \varphi_{F_1}(z) \le \frac{1}{2} \; or \; \varphi_{F_2}(z) \ge \varphi_{F_1}(z) \ge \frac{1}{2} \; then \; I_f(F_2) \le I_f(F_1).$

$(iv) If \; F \in FS(Z), then \; I_f(F) = I_f(F^C).$

Note: $I_{f(j)}(F_1, F_2) \; \forall \; j = 1,2, \ldots, 12 \; are \; interval \; entropies.$

The real function $I_{SM}: FS(Z) \times FS(Z) \longrightarrow R([0,1])$ is called interval similarity measure on $FS(Z)$, if I_{SM} has the following propositions:

(i) If $F \in CS(Z)$, then $I_{SM}(F, F^C) = [0,0]$.

(ii) If $F \in FS(Z)$, then $I_{SM}(F, F) = [1,1]$.

(iii) For $F_1, F_2 \in FS(Z)$, then $I_{SM}(F_1, F_2) = I_{SM}(F_2, F_1)$.

(iv) For $F_1, F_2, F_3 \in FS(Z)$, if $F_1 \subseteq F_2 \subseteq F_3$, then $I_{SM}(F_1, F_3) \leq I_{SM}(F_1, F_2)$ and $I_{SM}(F_1, F_3) \leq I_{SM}(F_2, F_3)$.

Note: $I_{SM(j)}(F_1, F_2)\ \forall\ j = 1,2,\ldots,10$ are interval similarity measures.

The real function $I_{DM}: FS(Z) \times FS(Z) \longrightarrow R([0,1])$ is called interval distance measure on $FS(Z)$, if I_{DM} has the following propositions:

(i) If $F \in CS(Z)$, then $I_{DM}(F, F^C) = [1,1]$.

(ii) If $F \in FS(Z)$, then $I_{DM}(F, F) = [0,0]$.

(iii) For $F_1, F_2 \in FS(Z)$, then $I_{DM}(F_1, F_2) = I_{DM}(F_2, F_1)$.

(iv) For $F_1, F_2, F_3 \in FS(Z)$, if $F_1 \subseteq F_2 \subseteq F_3$, then $I_{DM}(F_1, F_2) \leq I_{DM}(F_1, F_3)$ and $I_{DM}(F_2, F_3) \leq I_{DM}(F_1, F_3)$.

Note: $I_{DM(j)}(F_1, F_2)\ \forall\ j = 1,2,\ldots,10$ are interval distance measures.

The real function $I_{IM}: FS(Z) \times FS(Z) \longrightarrow R([0,1])$ is called interval inclusion measure on $FS(Z)$, if I_{IM} has the following propositions:

(i) $I_{IM}(Z, \Phi) = [0,0]$.

(ii) $I_{IM}(F_1, F_2) = [1,1] \Leftrightarrow F_1 \subseteq F_2$.

(iii) For $F_1, F_2, F_3 \in FS(Z)$, if $F_1 \subseteq F_2 \subseteq F_3$, then $I_{IM}(F_3, F_1) \leq I_{IM}(F_2, F_1)$ and $I_{IM}(F_3, F_1) \leq I_{IM}(F_3, F_2)$.

Note: $I_{IM(j)}(F_1, F_2)\ \forall\ j = 1,2,3,4$ are interval inclusion measure

Relationships between I_{SM}, I_{DM} and I_{IM}:

(i) If $I_f(FS)$ is interval entropy of fuzzy sets, $\forall\ F_1, F_2 \in FS(Z)$ and $\forall\ z \in Z$,

$$g_1(F_1, F_2)(z) = \frac{\varphi_{F_1}(z) \wedge \varphi_{F_2}(z)}{2\left(\varphi_{F_1}(z) \vee \varphi_{F_2}(z)\right)};\ g_2(F_1, F_2)(z) = \frac{1 - |\varphi_{F_1}(z) - \varphi_{F_2}(z)|}{2} \text{ and}$$

$$g_3(F_1, F_2)(z) = \frac{1 + |\varphi_{F_1}(z) - \varphi_{F_2}(z)|}{2} \text{ then}$$

(a) $I_f\left(g_j(F_1, F_2)\right)$ $(j = 1,2,3)$ are interval similarity measures of the fuzzy sets.

(b) $I_f{}^C\left(g_j(F_1, F_2)\right)$ $(j = 1,2,3)$ are interval distance measures of the fuzzy sets.

(c) $I_f\left(g_j(F_2, F_1 \cup F_2)\right)$ $(j = 1,2,3)$ are interval inclusion measures of the fuzzy sets.

(d) $I_f\left(g_j(F_1 \cap F_2, F_1)\right)$ $(j = 1,2,3)$ are interval inclusion measures of the fuzzy sets.

(ii) If $I_{DM}(FS)$ is interval distance measure of fuzzy sets, $\forall\ F_1, F_2 \in FS(Z), z \in Z$, then

(a) $I_{DM}{}^C(F_1, F_2)$ is interval similarity measure.

$(b) I_{DM}{}^{C}(F, F^{C})$ *is interval entropy.*
$(c) I_{DM}{}^{C}(F_2, F_1 \cup F_2), I_{DM}{}^{C}(F_1 \cap F_2, F_1)$ *are interval inclusion measures.*
$(iii) If\ I_{IM}(FS)$ *is interval inclusion measure of fuzzy sets,* $\forall F_1, F_2 \in FS(Z), z \in Z$*, then*
$(a) I_{IM}(F_1 \cup F_2, F_1 \cap F_2)$ *is interval similarity measure.*
$(b) I_{IM}(F \cup F^{C}, F \cap F^{C})$ *is interval entropy.*
$(c) I_{IM}{}^{C}(F_1 \cup F_2, F_1 \cap F_2)$ *is interval distance measure.*
In particular, it might be achieved that among the above different interval entropy measures, in general any two of the measures might be transformed within one another.

IV. APPLICATIONS OF FUZZY ENTROPY MEASURES

The fluffy set hypothesis has been used in a wide range of areas, including bioinformatics, discourse acknowledgment, highlight determination, fluffy airplane control, programmed control, navigation, sign and picture handling, and example acknowledgment. Numerous researchers have dedicated their research to advancing the fluffy set hypothesis and its ideas in order to improve its capacity and dexterity in dealing with vulnerability. For instance, various strategies are developed by numerous experts using summed fluffy disparity measures to consider various multi-standard dynamic issues. The various applications of the summed fluffy dissimilarity estimates have demonstrated the strength in the context of multi-rules dynamic issues and example acknowledgment issues. The use of fluffy set hypotheses and fluffy measures has led to the development of numerous new approaches to these kinds of problems, which raise the broad concern of vulnerabilities.
Examples:

Pattern Recognition Problems: For determining the different applications of the generalized fuzzy divergence measures in these problems. Let us suppose there are given $'i'$ known patterns $p_1, p, p_3, \dots, p_i$ which have different classifications $c_1, c_2, c_3, \dots, c_i$ respectively. Let these patterns are described through the resulting fuzzy sets in the universe of discourse $Y = \{y_1, y_2, \dots, y_j\}$:

$$p_m = \{(y_n, \varphi_p(y_n))/y_n \in Y\}, where\ m = 1,2, \dots, i\ and\ n = 1,2, \dots, j.$$

Suppose that there is an unknown pattern q, described by the fuzzy set

$$q_m = \{(y_n, \varphi_{q_m}(y_n))/y_n \in Y\}.$$

Now, use the principle of minimum divergence measure between the fuzzy sets to classify q by one of the classes $c_1, c_2, c_3, \dots, c_i$, then the measure of selecting $q\ to\ c_{o^*}$ is represented as:

$$c_{o^*} = arg \min_{o}\{D(p_o, q)\}.$$

By using this method, given pattern could be accepted as the best class could be selected. For these types of problems, it is an efficient application of the principle of minimum divergence measure [91].

Decision-making problems: These problems are used to find the favorite choice from the group of the suitable options. Let us suppose that a firm 'F' needs to cover 'm' strategies $s_1, s_2, \dots, s_m$ for its target and the various strategies have the distinct degree of performance if the profit related with it varies. Suppose that $\{i_1, i_2, \dots, i_n\}$ be the set of options and the fuzzy set X stands for the degree of performance of a selective strategy with an attire profit. Hence, $X = \{(X, \varphi_X(s_i)) : i = 1,2,..m\}$.

Now, suppose that I_j be a fuzzy set stands for the degree of performance of a selective strategy if it is achieved with input I_j.

$$I_j = \left\{\left(I_j, \varphi_{I_j}(s_i)\right) : i = 1,2,..m\right\} \; where \; j = 1,2,\dots,n.$$

In the fuzzy divergence measure by taking$F_1 = X \; and \; F_2 = I_j$, a new parametric generalized fuzzy exponential measure $I_{E_\beta}(X, I_j)$ is assigned.

Then, the most effective I_j is determined as $Min\left\{I_{E_\beta}(X, I_j)\right\}_{\substack{1 \le j \le n \\ 0 < \beta \le 1/2}}$.

Also, let $I_l (1 \le l \le n)$ resolves the minimum value of $\left\{I_{E_\beta}(X, I_j)\right\}_{0 < \beta \le 0.5}$.With this I_l find $Max\left\{\varphi_{I_j}(s_i)\right\}_{1 \le i \le m}$ so that it correlate to $s_p, 1 \le p \le m$.

Thus, when the strategy s_p is achieved with a profit (input)I_l, then the firm will contact with its target in the better profit-impressive way.

V.CONCLUSION

The fundamental definitions, properties, and examples of the various fluffy entropy proportions discussed in this paper are outlined. Additionally, we have examined their relatives. It has been observed that fluffiness is one approach to the enrollment of a component of a set, and it may be unilateral. However, by the end of the fluffy incentive for each component of the set, the diverse range is rational. Additionally, this paper provides specific examples of various proportions of fluffy data. As a result of the development of academic entropy, specific structures import fluffy entropy as a proportion of the fluffy set data. The cases of the all-entropy proportion of fluffy sets are also suggested, as are the fundamental sayings, various properties, and examples. It has been established that any two of the aforementioned stretch entropy measures can be adjusted toward one another. Additionally, we observe that the various applications employ distinct entropy meanings. We continue to collect and observe the various regions of the various uses of the specific fluffy entropies, and we will undoubtedly observe its preservation in the future before it is finished.

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VII.CONFLICT OF INTEREST

NIL

VIII. REFERENCES

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