

## Construction of Six Sigma Control Chart with Waiting Time for (M/M/S) Queueing Model

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### ABSTRACT

Buyer concluded quality in covering framework can be accomplished by making control outlines for standard holding up time and preparing control limits for something as per an overall perspective muddled from use time. This examination article joins the improvement of six sigma control diagram with the assistance of cycle limit concerning holding up time what's more it gives appropriate tables to M/M/S lining model.

**Keyword:** Control frame, Queueing model, Cycle cutoff and Six sigma.

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### I. INTRODUCTION

Lines are for the most part around seen at banks, emergency focuses, cafeterias, rail course saving counters, work trade, film theaters, etc. If all else fails, lines structure when there is a lot of interest on the working environments with the objective that there is a flood of holding up time or lacking number of alliance facilities. The fixing associate with the time spent by clients to get to affiliations is changing into a fundamental issue to overwhelm networks, Poongodi and Muthulakshmi (2013). Working with a monstrous load of affiliation ability to work an improvement winds around foolishly cost to overwhelm networks and not offering mind blowing assist with restricting makes crazy pausing and cost to clients. An ideal assistance level with canning be accomplished by needing to hold up times and making staffing. At any rate great evaluation exists on queueing structures, moreover little thought has been given to the certain checking of utilitarian execution, which will help with accomplishing more significant activities (Pukazhenthhi and Poornima, 2018).

Montgomery (2005) has proposed various inspirations driving control graphs in guarantying quality in party affiliations. Subsequently the evaluation of time spent in the strategy by the control outline structure is reasonable here. By considering the central three minutes Shore (2000) made control outline for rash line length, N of M/M/1 covering model. Khaparde and Dhabe (2010) remained mindful of the control outline including system for weighted contrast for conflicting line length N for M/M/1 queuing model and took withdrew holding up time in plan of M/M/1 covering model using control outline. In this assessment article it is considered to attract the six sigma control outline for holding up time, W of M/M/s queuing model.

## **II. CONCEPT OF SIX SIGMA**

Six sigma at many firm plans a degree of central worth that put everything at risk. The term six sigma, contemplating everything, propose a goliath heap of major worth control instruments that affiliations can use to keep away from move away and further urge cycles to help to raise their benefits. A researcher working at Motorola empowered this six sigma in 1980s. Six sigma is a quantifiable and information working cycle that works by confining cutoff goofs or leaves. It stays mindful of cycle term stimulates while hacking down the party mishapenness to something like 3.4 events per million units or occasions. This shows that an oversight could happen with a six-standard deviation occasion from the mean considering the way that just 3.4 out of 1,000,000 occasions along a ringer bend will lie past six standard deviations.

## **III. MODEL DESCRIPTION FOR M/M/S**

M/M/s model has' servers formed in in distinguishable where the help time at each counter is all over something basically indistinct and sees the primary rule. A client can get into any of the free counters for alliance. The improvement has whoever needs it most will impact the drawn out length (FCFS) line discipline with clear end. The clients show up in a Poisson dispersal with mean appearance rate  $\lambda$  and get association with a mean help rate  $\mu$ .

### ***a. Control chart for waiting time (W) for M/M/s Model***

By approximating the assessment possible by an all-around standard dissipating the Shewhart type control outlines are made. Correspondingly the restrictions of the control outline (Poongodi and Muthulakshmi, 2013) are given by

$$UCL=E(u)+3\sqrt{V(u)}$$

$$CL=E(u)$$

$$LCL=E(u)-3\sqrt{V(u)}$$

The parameters of the control chart for waiting time of the customer in the system for M/M/s queuing model is given as,

$$UCL = \frac{1}{\mu_s} \left\{ \left( \frac{(s\rho)^s}{s!s(1-\rho)^2} Pr_0 + 1 \right) + 3 \left\{ \sqrt{2 \left( 1 + \frac{(\rho s)^s (\mu_s(s+1) - \lambda_A)}{\mu_s s! s^2 (1-\rho)^3} Pr_0 \right) - \left( 1 + \frac{(\rho s)^s}{s!s(1-\rho)^2} Pr_0 \right)^2} \right\} \right\}$$

$$CL = \frac{(s\rho)^s}{s!s\mu_s(1-\rho)^2} Pr_0 + \frac{1}{\mu_s}$$

$$LCL = \frac{1}{\mu_s} \left\{ \left( \frac{(s\rho)^s}{s!s(1-\rho)^2} Pr_0 + 1 \right) - 3 \left\{ \sqrt{2 \left( 1 + \frac{(\rho s)^s (\mu_s(s+1) - \lambda_A)}{\mu_s s! s^2 (1-\rho)^3} Pr_0 \right) - \left( 1 + \frac{(\rho s)^s}{s!s(1-\rho)^2} Pr_0 \right)^2} \right\} \right\}$$

**b. Six sigma based waiting time control chart for M/M/s Model**

For a specified TL and  $C_{PC}$  of the process (Radhakrishnan and Balamurugan, 2012), the value of  $\sigma$  (termed as  $\sigma_{6\sigma}$ ) is calculated from  $C_{PC} = \frac{TL}{6\sigma}$  using a computer program for various combinations of TL and  $C_{PC}$

$$UCL = \left( \frac{(s\rho)^s}{s!s\mu_s(1-\rho)^2} Pr_0 + \frac{1}{\mu_s} \right) + 4.831\sigma_{6\sigma}$$

$$CL = \frac{1}{\mu_s} + \frac{(s\rho)^s}{s!s\mu_s(1-\rho)^2} Pr_0$$

$$LCL = \left( \frac{(s\rho)^s}{s!s\mu_s(1-\rho)^2} Pr_0 + \frac{1}{\mu_s} \right) - 4.831\sigma_{6\sigma}$$

**IV. PRACTICAL APPLICATION**

The numerical illustration with certain selected values of  $\lambda_A$  and  $\mu_s$  is used in constructing the six sigma control chart for mean waiting time based on M/M/s model as follows:

From the below Table-1, it shows that the filling in appearance rate nearby clear assistance rate, the standard holding up time and the normal farthest major of holding up time is thusly extended and it is portrayed in the Figure - 1. The control limits stretch (CLI) of six sigma is more unassuming than past what many would think about conceivable time span. Obviously the wearisome way of thinking isn't in exceptional quality absolutely precisely true to form, other than a change and improvement is standard in the queuing structure.

Table 1: Shewhart control chart and six sigma control chart for parameters  $\mu_s = 3$  and  $s=2$

Arrival rate ( $\lambda_A$ )	Service rate ( $\mu_S$ )	Number of service channel (s)	Busy time ( $\rho$ )	$Pr_0$	Shewhart control chart			Six sigma control chart ( $\sigma_{6\sigma} = 0.0055$ )	
					LCL	CL	UCL	LCL	UCL
0.20	3	2	0.0333	0.9335	0.3335	0.0000	0.3337	1.3343	0.3071
0.40	3	2	0.0667	0.8678	0.3341	0.0000	0.3348	1.3372	0.3082
0.60	3	2	0.1000	0.8036	0.3351	0.0000	0.3366	1.3421	0.3101
0.80	3	2	0.1333	0.7414	0.3366	0.0000	0.3392	1.3491	0.3126
1.00	3	2	0.1667	0.6818	0.3386	0.0000	0.3424	1.3583	0.3159
1.20	3	2	0.2000	0.6250	0.3411	0.0000	0.3464	1.3697	0.3198
1.40	3	2	0.2333	0.5712	0.3442	0.0000	0.3510	1.3835	0.3244
1.60	3	2	0.2667	0.5205	0.3479	0.0000	0.3563	1.3999	0.3297
1.80	3	2	0.3000	0.4730	0.3522	0.0000	0.3623	1.4190	0.3357
2.00	3	2	0.3333	0.4286	0.3573	0.0000	0.3690	1.4411	0.3425
2.20	3	2	0.3667	0.3872	0.3633	0.0000	0.3766	1.4666	0.3500
2.40	3	2	0.4000	0.3488	0.3703	0.0000	0.3850	1.4960	0.3584
2.60	3	2	0.4333	0.3133	0.3786	0.0000	0.3944	1.5301	0.3678
2.80	3	2	0.4667	0.2804	0.3882	0.0000	0.4049	1.5695	0.3783
3.00	3	2	0.5000	0.2500	0.3997	0.0000	0.4167	1.6156	0.3901

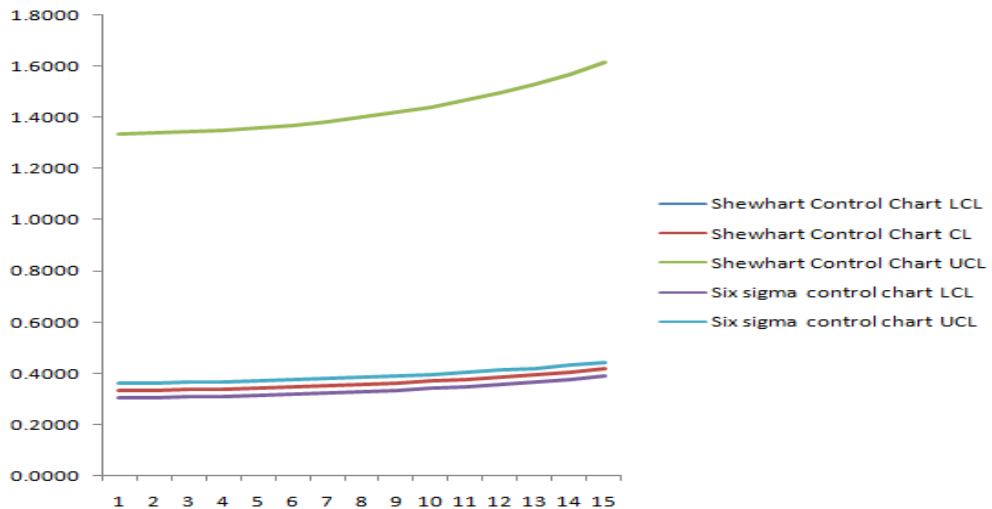


Figure 1: Shewhart control chart and six sigma control chart for parameters  $\mu_s=3$  and  $s=2$

Table 2: Shewhart control chart and six sigma control chart for parameters  $\mu_s=3$  and  $s=3$

Arrival rate ( $\lambda_A$ )	Service rate ( $\mu_S$ )	Number of service channel (s)	Busy time ( $\rho$ )	$P_0$	Shewhart Control Chart			Control Chart using process capability ( $\sigma_{6\sigma} = 0.0030$ )	
					LCL	CL	UCL	LCL	UCL
0.20	3	3	0.0222	1.0689	0.3333	0.0000	0.3333	1.3333	0.3188
0.40	3	3	0.0444	1.1426	0.3334	0.0000	0.3334	1.3334	0.3189
0.60	3	3	0.0667	1.2214	0.3334	0.0000	0.3335	1.3338	0.3190
0.80	3	3	0.0889	1.3057	0.3335	0.0000	0.3339	1.3345	0.3194
1.00	3	3	0.1111	1.3959	0.3338	0.0000	0.3345	1.3359	0.3201
1.20	3	3	0.1333	1.4925	0.3342	0.0000	0.3357	1.3384	0.3212
1.40	3	3	0.1556	1.5960	0.3350	0.0000	0.3375	1.3424	0.3231
1.60	3	3	0.1778	1.7072	0.3361	0.0000	0.3404	1.3488	0.3259
1.80	3	3	0.2000	1.8269	0.3379	0.0000	0.3448	1.3583	0.3303
2.00	3	3	0.2222	1.9562	0.3404	0.0000	0.3511	1.3722	0.3366
2.20	3	3	0.2444	2.0962	0.3439	0.0000	0.3602	1.3919	0.3457
2.40	3	3	0.2667	2.2485	0.3486	0.0000	0.3730	1.4189	0.3585
2.60	3	3	0.2889	2.4150	0.3547	0.0000	0.3909	1.4549	0.3764
2.80	3	3	0.3111	2.5979	0.3618	0.0000	0.4158	1.5012	0.4013
3.00	3	3	0.3333	2.8000	0.3693	0.0000	0.4500	1.5579	0.4355

From the above Table-2, it shows that the filling in appearance rate nearby clear help rate, the typical holding up time and the standard most clear need of holding up time is other than expanded and it is begun from the Figure-2. Particularly far range (CL) of six sigma is more unassuming than past what many would think about conceivable time frame.

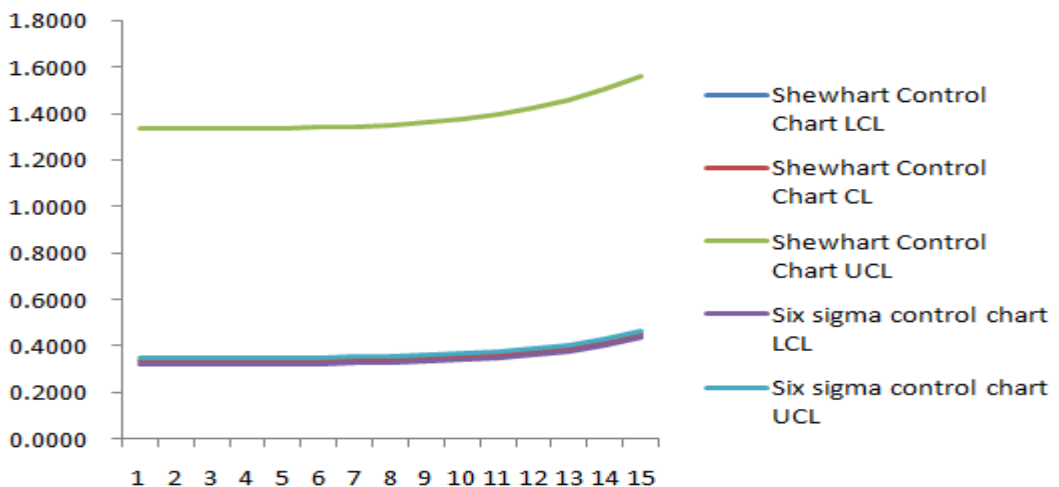
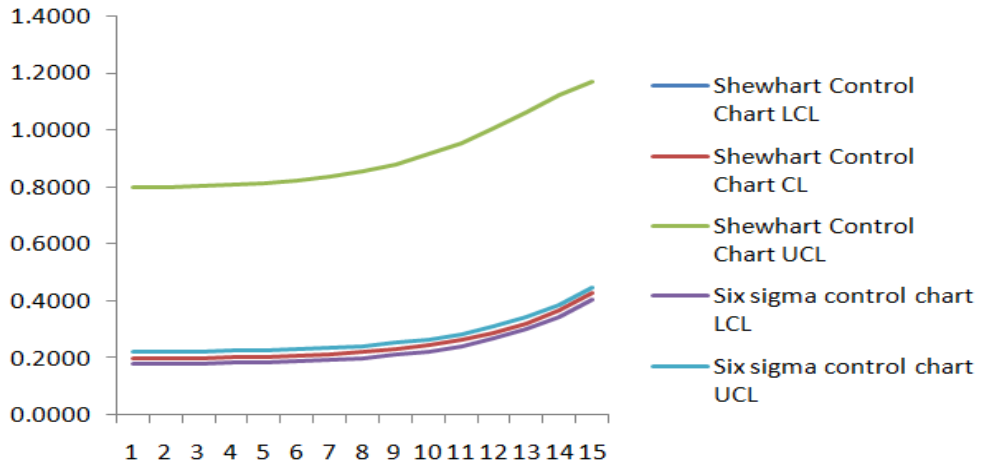


Figure 2: Shewhart control chart and six sigma control chart for parameters  $\mu=3$  and  $s=3$

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**Figure 3: Shewhart control chart and control chart using process capability for parameters  $\mu=5$  and  $s=2$**

**Table 3: Shewhart control chart and six sigma control chart for parameters  $\mu=5$  and  $s=2$**

Arrival rate ( $\lambda_A$ )	Service rate ( $\mu_S$ )	Number of service channel (s)	Busy time ( $\rho$ )	$P_0$	Shewhart Control Chart			Control Chart using process capability ( $\sigma_{6\sigma} = 0.0043$ )	
					LCL	CL	UCL	LCL	UCL
0.25	5	2	0.0250	1.0538	0.2001	0.0000	0.2001	0.8004	0.1794
0.50	5	2	0.0500	1.1157	0.2003	0.0000	0.2006	0.8016	0.1798
0.75	5	2	0.0750	1.1863	0.2008	0.0000	0.2016	0.8041	0.1808
1.00	5	2	0.1000	1.2663	0.2017	0.0000	0.2031	0.8082	0.1824
1.25	5	2	0.1250	1.3565	0.2031	0.0000	0.2055	0.8147	0.1848
1.50	5	2	0.1500	1.4580	0.2051	0.0000	0.2091	0.8243	0.1883
1.75	5	2	0.1750	1.5720	0.2079	0.0000	0.2141	0.8379	0.1934
2.00	5	2	0.2000	1.6998	0.2118	0.0000	0.2212	0.8567	0.2005
2.25	5	2	0.2250	1.8428	0.2169	0.0000	0.2311	0.8818	0.2103
2.50	5	2	0.2500	2.0030	0.2234	0.0000	0.2445	0.9146	0.2237
2.75	5	2	0.2750	2.1822	0.2310	0.0000	0.2628	0.9559	0.2420
3.00	5	2	0.3000	2.3829	0.2395	0.0000	0.2875	1.0060	0.2668
3.25	5	2	0.3250	2.6075	0.2474	0.0000	0.3209	1.0631	0.3001
3.50	5	2	0.3500	2.8592	0.2520	0.0000	0.3658	1.1219	0.3450
3.75	5	2	0.3750	3.1412	0.2474	0.0000	0.4262	1.1685	0.4054

From the above Table-3, it shows that the filling in appearance rate with predictable assistance with rating fosters the standard holding time and the standard most clear deterrent of holding up time and it is started from the Figure-3 that amazingly far time span utilizing process limit is more unassuming than past what many would think about conceivable time span. Other than the examination between Figures-1, 2 and 3 redesigns in number of servers' flood the standard

holding up time and lesser party in the six sigma control outline considered the relentless control frame.

## **V. CONCLUSION**

The proposed six sigma based control outline leads better appeared contrastingly as shown by the expected control diagrams and incredibly far range (CLI) of proposed control chart is more unassuming than past what many would consider conceivable time period. Obviously the improvement of the arrangement is denied than the interest considering the six sigma control chart. The proposed control frame for M/M/S model will expand the satisfaction and mental strength of the purchasers.

## **VI. REFERENCES**

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