

Applications of Six Sigma Control Chart in Queuing System M[X]/M/1

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ABSTRACT

A cycle is in managed, thinking about beyond experience. It will continuously be the same old the manner that the association will move (internal limits) beginning right here till an uncommonly brutal time span to come. The manage define framework can be used to take a gander on the preserving time of the customers withinside the development to control the affiliations and the strong presentation of stresses. In this evaluation article, the development of the six-sigma manage body for typically round popular development duration is proposed and furnishes realistic tables with numerical effects for M [x]/M/1 queuing shape while the % length follows a numerical new flip of occasions.

Keyword: Control define, numerical spread queuing are examples of keywords. model, cycle cutoff, and 6 sigma.

I. INTRODUCTION

Lining speculation, as a bit of attempted studies, can assist customers discover ways to upload as much as affordable and unessential value paintings system systems withinside the area of commercial enterprise. Lining speculation changed into finished and morally legitimised for the primary time withinside the 20th century to acquire hits and forestall troubles (Pukazhenth and Poornima, 2018). In this manner, it is a begin and cease to a certainly taken into consideration, continuously notion-upsetting idea. Today, this assessment is commonly utilized by affiliations like Airtel, Walmart, AT & T, and Verizon, as proven with the aid of using a popular factor of view, greater to set them up for destiny visitors certainly early. (www.analyticsvidhya.com). Lining speculation, as a assured application, covers a huge variety of

purposes, for example, how to devise tremendous customer assist, replace visitors stream, and reduce off shipping orders from a warehouse. Speculation works with troubles that be part of lines (www.human beings.brunel.ac.uk). The popular fashions may be:

- ✓ banks, stores, and retail outlets—final aware of a few shape of affiliation manage
- ✓ Workstations: Holding tight for a response
- ✓ For example, withinside the manufacture of clothing
- ✓ public transportation-saving a diploma of manage for a car or train

This examine protected robust updates for first-class manage to increase closer to the virtuoso of a sewage remedy station, Fabio Orssatto et al. (2014). Working with an sizeable shop of hobby potential to make an development provides a stupid value for virtuoso obsessions, and now no longer providing clean assist with confining makes hopeless stops and prices to customers. An best assist degree with capping may be accomplished with the aid of using desiring to maintain up instances and development staffing. At any rate, overwhelming appraisal exists on queueing systems, and normal, little notion has been given to the quantifiable checking of beast execution, in an effort to help in attaining greater realistic exercises.

II. Concepts and terminologies

a) Arrival pattern

The look plan tells how the devices seem and be part of the brand new development. The wellspring of the presence of the devices might not be absolutely settled. The look of the unit can be both uninhibitedly or in a celebration. The look plan is incessantly displayed on an surprisingly essential degree, in a bewildered manner as the overall popular variety of appearances in step with unit time.

b) Service pattern

The approach shape tells us how the assistance is proposed for the appearances. The arrival of the devices can be served both uninhibitedly or in a captivating occasion. The time anticipated for serving as an arrival unit is referred to as association time. The assist define with perseverance can be both constant or non-constant regarding time and country and can be based or loose regarding how a good deal customers save, a valid diploma of manage for the affiliation.

c) Queue discipline

Line discipline, which proposes plans for acquiring customers with the aid of using retaining a excessive degree of manage over the road of hobby, The maximum difficult line disciplines in phrases of customer appearances withinside the motion are people who first start matters out of service (FCFS) and remaining start matters out of service (LCFS). Clients are directly free of their appearances to take part withinside the development referred to as "courting in clashing proposing" (SIRO).

d) Upper specification limit (USL)

It is shocking, commonly speaking, to look human beings speaking round being proven with the aid of using the manufacturer for a cycle or element to have an very well display.

e) Lower specification limit (LSL)

It is the humblest reference proven with the aid of using the manufacturer for a cycle or element to have an very well display.

f) Tolerance level (TL)

It is a quantifiable attain internal which, completely degree, a named degree of a proven % falls. It is the department among USL and LSL, $TL = USL - LSL$.

g) Process capability (C_p)

Process restrict studies exhibits the surrendered, yielded, viable, inescapable effect of an in-manage courting with the beyond, which many might remember viable with the aid of using the usage of restrict records (Montgomery, 2010).

The rule systems for 6 sigma, which can be utilized in exclusive enterprise conditions, are as proven through the following:

DMAIC is a facts-pushed method for attracting and inspiring the norm to go to with ranges of development and cycles. It is a vast development for managed alternate, the pioneers. The diverse activities of DMAIC are recorded, and every degree joins instruments and tries to assist with locating the final approach.

- Depict the problem and set ahead the desires of the endeavor.
- Measure the exclusive portions of the same old cycle definitely.
- Take a gander on the facts to look the beast bowing withinside the information.
- Work at the given endeavor.
- Control over how the cycle is done starts right here and maintains till an notable possibility stretches ahead.

Six sigma is a sure, certain, and facts certain technique that works through isolating restriction bobbles or leaves. It specializes in technique variety healing at the same time as lowering celebration mutilations to something like 3.4 activities in line with million devices or occasions. This proposes that a surprise throughout takes place with a six-preferred deviation occasion from the mean, thinking about the manner that handiest 3.4 out of a million occasions alongside a hoop flip might fall beyond six preferred deviations.

As such, the appraisal of time spent at the development through the manage define shape is assured right here. Shore (2000) advanced a manage define for clashing line period, N of M/M/1, and included model. In this assessment article, it's far taken into consideration to attract on Shewhart's (1931) six-sigma primarily based totally manage body for M [x]/M/1 queuing shape whilst the % length follows numerical dispersal.

III. Model description for M/M/s

Consider queuing model with a single server in which the arrivals occur in batches according to Poisson process with rate λ_A . Here, the batch size X is a random variable with $\Pr(X=k)=C_k$, $k=1, 2, \dots$. Also the customers are served one by one and the service time is exponentially distributed at the rate μ_s .

a. Control chart for average system length (N_s)

Shewhart (1931) type of control charts are constructed by approximating the statistic under consideration by a normal distribution. The parameters of the control chart are as follows

$$UCL = E(N_s) + 3\sqrt{V(N_s)}$$

$$CL = E(N_s)$$

$$LCL = E(N_s) - 3\sqrt{V(N_s)}$$

The parameters of the control chart (Poongodi and Muthulakshmi, 2013) for M/M/1 queueing model using $E(N_s)$ and $V(N_s)$, We get

$$LCL = \frac{\rho}{(1-\alpha)(1-\rho)} - 3\frac{\sqrt{\rho[1+\alpha(1-\rho)]}}{(1-\alpha)(1-\rho)}$$

$$CL = \frac{\rho}{(1-\alpha)(1-\rho)}$$

$$UCL = \frac{\rho}{(1-\alpha)(1-\rho)} + 3\frac{\sqrt{\rho[1+\alpha(1-\rho)]}}{(1-\alpha)(1-\rho)}$$

b. Control chart for average system length (N_s) using process capability (C_p)

For a specified TL and C_{PC} of the process (Radhakrishnan and Balamurugan, 2010), the value of σ (termed as σ_Q) is calculated from $C_{PC} = (TL/6\sigma)$ using a computer program for various combinations of TL and C_{PC} .

$$LCL_Q = \frac{\rho}{(1-\alpha)(1-\rho)} - 3\sigma_Q$$

$$CL_Q = \frac{\rho}{(1-\alpha)(1-\rho)}$$

$$UCL_Q = \frac{\rho}{(1-\alpha)(1-\rho)} + 3\sigma_Q$$

IV. Practical application

The following Table-1 gives the traffic intensity and the control chart parameters for average system length for $\lambda_A = 3$, $\mu_S = 7$ and $\alpha = 0.01$ to 0.10, we get

Table 1: Control limits for average system length for $\lambda_A=3$, $\mu_S=7$ and $\alpha=0.01$ to 0.10

Arrival rate (λ_A)	Service rate (μ_S)	Number of customers in arriving batch (α)	Busy time (ρ)	Standard deviation (σ)	Control limits for M ^[X] /M/1 Model			Six sigma control chart for M ^[X] /M/1 Model ($\sigma_{6\sigma} = 0.036$)	
					LCL	CL	UCL	LCL	UCL
3	7	0.01	0.433	1.175	-2.755	0.771	4.297	0.597	0.945
		0.02	0.437	1.206	-2.825	0.793	4.411	0.619	0.967
		0.03	0.442	1.238	-2.898	0.816	4.530	0.642	0.990
		0.04	0.446	1.271	-2.973	0.840	4.653	0.666	1.014
		0.05	0.451	1.306	-3.052	0.865	4.782	0.691	1.039
		0.06	0.456	1.342	-3.133	0.891	4.916	0.718	1.065
		0.07	0.461	1.379	-3.218	0.919	5.056	0.745	1.093
		0.08	0.466	1.418	-3.307	0.948	5.203	0.774	1.122
		0.09	0.471	1.459	-3.399	0.978	5.355	0.804	1.152
	0.1	0.476	1.502	-3.495	1.010	5.515	0.836	1.184	

From the above Table 1, it indicates that at the same time as attracting the chance of displaying up customers in a celebration with a clean look rate ($\lambda_A=3$) and a push closer to rate ($\mu_S=7$), then, at that point, the same old period of the brand new flip of occasions and the same old farthest endpoints' greater makes. This is proven in Figure 1, in particular withinside the some distance term sigma manage body, that's greater unassuming than what many might consider a probable stretch of time. Clearly, the close machine is not of extraordinary quality, unequivocally, definitely convincingly, legitimately, and precisely as predicted, aside from a alternate and development being preferred withinside the queueing shape.

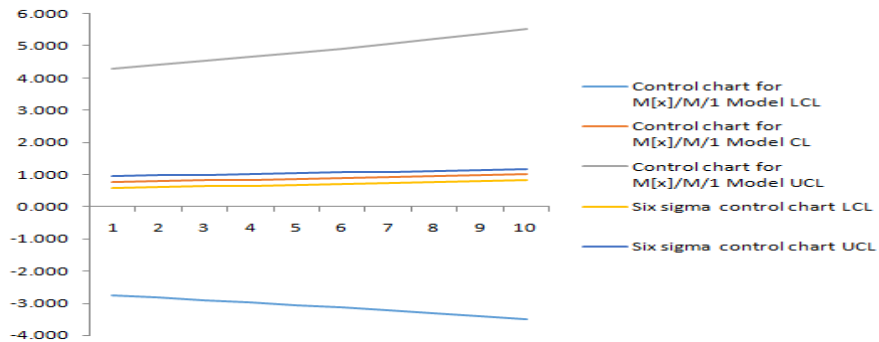


Figure 1: Control limits for average system length for $\lambda_A=3$, $\mu_S=7$ and $\alpha=0.01$ to 0.10

By seeing beneathneath Table-2, whilst there may be a filling in what number of customers display up load with clean look ($\lambda_A=3$) and plot rate ($\mu_S=14$) there may be this kind of development withinside the goliath development period and the same old top limits. It is obvious from Figure 2 that the six sigma manage body completed higher and confirmed up contrastingly just like the Shewhart body.

Table 2: Control limits for average system length for $\lambda_A=3, \mu_S=14$ and $\alpha=0.11$ to 0.20

Arrival rate (λ_A)	Service rate (μ_S)	Number of customers in arriving batch (α)	Busy time (ρ)	Standard deviation (σ)	Control limits for M ^[X] /M/1 Model			Six sigma control chart for M ^[X] /M/1 Model ($\sigma_{6\sigma} = 0.021$)	
					LCL	CL	UCL	LCL	UCL
3	14	0.11	0.241	0.756	-1.911	0.356	2.624	0.255	0.458
		0.12	0.244	0.774	-1.957	0.366	2.688	0.264	0.467
		0.13	0.246	0.793	-2.004	0.376	2.755	0.274	0.477
		0.14	0.249	0.813	-2.052	0.386	2.824	0.284	0.487
		0.15	0.252	0.833	-2.102	0.397	2.895	0.295	0.498
		0.16	0.255	0.854	-2.154	0.408	2.970	0.306	0.509
		0.17	0.258	0.876	-2.208	0.419	3.046	0.318	0.521
		0.18	0.261	0.898	-2.264	0.431	3.126	0.330	0.533
		0.19	0.265	0.922	-2.321	0.444	3.209	0.343	0.546
		0.2	0.268	0.946	-2.381	0.457	3.296	0.356	0.559

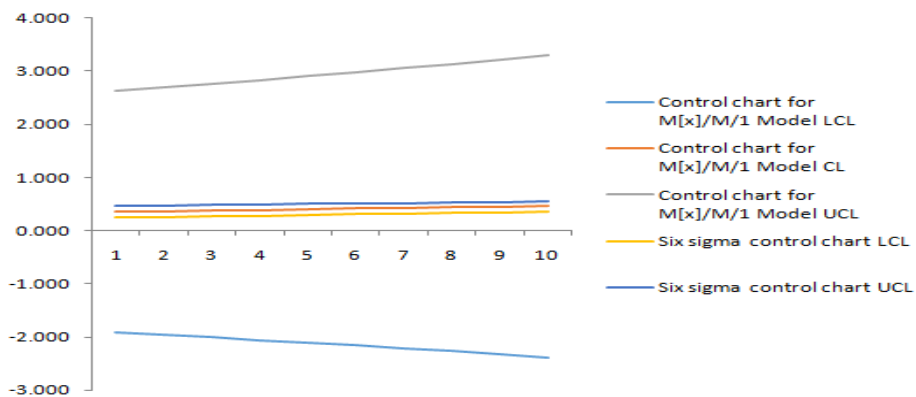


Figure 2: Control limits for average system length for $\lambda_A=3, \mu_S=14$ and $\alpha=0.11$ to 0.20

The following Table-3 gives the traffic intensity and the control chart parameters for average system length for $\lambda_A=3, \mu_S=10$ and $\alpha=0.01$ to 0.10 , we get

Table 3: Control limits for average system length for $\lambda_A=3, \mu_S=10$ and $\alpha=0.01$ to 0.10

Arrival rate (λ_A)	Service rate (μ_S)	Number of customers in arriving batch (α)	Busy time (ρ)	Standard deviation (σ)	Control limits for M ^[X] /M/1 Model			Six sigma control chart for M ^[X] /M/1 Model ($\sigma_{6\sigma} = 0.063$)	
					LCL	CL	UCL	LCL	UCL
3	10	0.01	0.303	0.801	-1.963	0.439	2.841	0.135	0.744
		0.02	0.408	1.108	-2.620	0.704	4.028	0.399	1.008
		0.03	0.412	1.136	-2.686	0.723	4.133	0.419	1.028
		0.04	0.417	1.166	-2.754	0.744	4.242	0.440	1.048
		0.05	0.421	1.197	-2.825	0.766	4.356	0.461	1.070
		0.06	0.426	1.229	-2.898	0.788	4.474	0.484	1.092
		0.07	0.430	1.262	-2.974	0.812	4.597	0.507	1.116
		0.08	0.435	1.296	-3.053	0.836	4.725	0.532	1.140
		0.09	0.440	1.332	-3.135	0.862	4.859	0.558	1.166
		0.1	0.444	1.370	-3.221	0.889	4.998	0.585	1.193

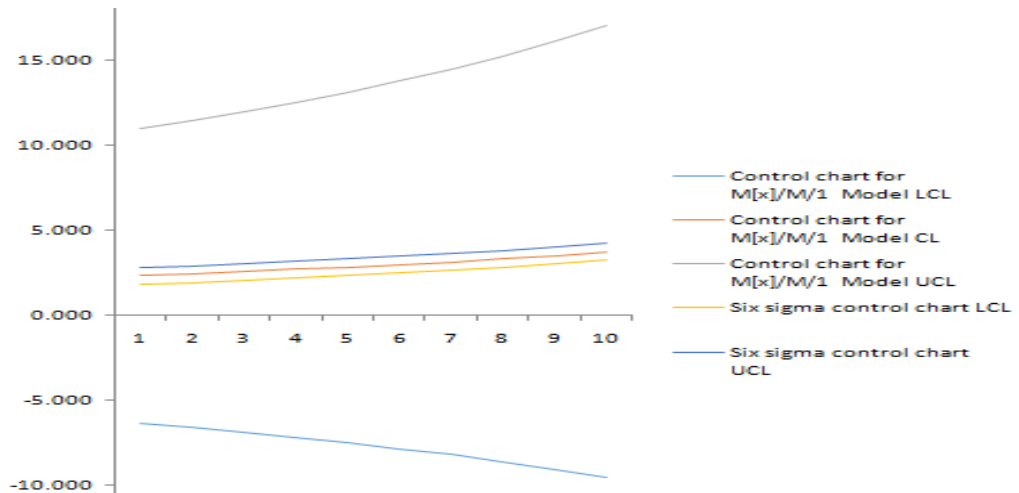


Figure 3: Control limits for average system length for $\lambda_A=3, \mu_S=10$ and $\alpha=0.01$ to 0.10

Based at the numerical consequences received in Table-3, for a regular fee of $=3, =five$ and an boom withinside the quantity of clients arriving in a batch, we discover an boom withinside the common machine period and the predicted top limits. Figure four well-known shows that the manage restriction c language of 3σ the use of technique functionality is smaller than the manage restriction c language of Shewhart.

V. CONCLUSION

In this evaluation article, it's far proposed an basic touchy document among Shewhart's manage plan and the six-sigma manage body for a clean approach period that wires exclusive look rates, credit score charges, and the customers' look to the celebration to keep away from the customers' mindless preserving up time withinside the line. The six sigma manage chart has greater capability than the stable manage body. As a result, the proposed body is saved in thoughts and activates to make monster movements to shorten the preserving time.

VI. REFERENCES

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