

Fuzzy Matrix Games with Intuitionistic Fuzzy Goals and Intuitionistic Fuzzy Linear Programming Duality

Priyanka Thakur¹, Shiv. K. Sharma²

*^{1,2}Department of Mathematics (UIS)
Chandigarh University Gharuan, Mohali, 140413, India*

ABSTRACT

In this paper a two person zero sum matrix game with fuzzy goals (TPZSMGFG) or constraints that can be introduced by (Atanassov's) Intuitionistic fuzzy sets. Intuitionistic fuzzy set (IFS) is equal to two crisp linear programming problems (CLPP) for two players which establish a primal-dual problem in the sense of linear programming duality in Intuitionistic fuzzy situation.

Keyword: Fuzzy payoffs, Fuzzy duality, Fuzzy Inequality, TPZSM, Fuzzy optimization

I. INTRODUCTION

LPP and fuzzy matrix games have studied a literature e.g. C. R. Bector et al. (2004), Nishizaki and Sakawa (2001), A. Aggarwal et.al (2012) and more references cited therein. Fuzzy matrix has many approaches like to model the medical diagnostic process and decision making process. Fuzzy sets are drafted to manage above-mentioned doubts by assigning a degree called degree of membership, by which an object belong to the set. When the same set and degree of membership does not belong to the each other then it is taken as minus one to the belongingness degree. So it is called non-belongingness degree. In genuine difficulties there are not only the degree of belongingness is known but also the degree of non-belongingness is familiar. Suppose, when we calculate a product then it could be 'good' and 'bad'. The argument on the product are evaluating on the basis of goodness or badness. The argument in the favor of good then it is degree of membership and if the argument is in the favor of bad then it is the degree of non-belongingness.

Atanassov (1986) suggested a fascinating generalization of fuzzy sets by capturing the behavior of human and it is known as Intuitionistic fuzzy set (IFS). IFS can be defined by the membership function, in which there is belongingness degree and non-belongingness degree.

The membership function can be defined from the universal set and the addition of both the degrees is less than one or equal to one and greater than zero or equal to zero. Intuitionistic fuzzy set plays great role in research and have wide applications. We refer many research Vlachos and Sergiadis (2007), Szmidt and Kacprzyk (1996), De et al. (2001) and many other references. Many set operations were explained by Atanassov (1986, 1989, 1994) on IFS. Li (2005) suggested an effective method on multi attributing decision making problems and techniques by using IFS. The characteristic of IFS refer by Dubois et al. (2005) and many more. After that IFS now called as “Atanassov’s I- fuzzy sets” or simple “I- fuzzy sets”. Bector et al. (2004) result shows the crisp game theory for fuzzy games. Compos (1989) worked earlier on fuzzy matrix games and develop techniques for explaining games by ranking function.

IFS has many applications so IFS provide an area of research and studies like decision-making or decision support system, academic career of the students, root type in image processing, sociometry, choice of discipline of study , medical diagnosis, medicines etc.

II. PRELIMINARIES

Let E be a universal set. An (IFS)X in E is defined as

$$X = \{(t, \mu_X(t), \nu_X(t)) \mid t \in E\}$$

here $\mu_X: E \rightarrow [0,1]$ and $\nu_X: E \rightarrow [0,1]$ defines the belongingness degree and the non-belongingness degree of function of an element $t \in E$ to the set X with $0 \leq \mu_X(t) + \nu_X(t) \leq 1$.

For two IFS X and Y in E, the union and intersection of IFS are defined as

$$X \cup Y = \{(t, \max\{\mu_X(t), \mu_Y(t)\}, \min\{\nu_X(t), \nu_Y(t)\}) \mid t \in E\},$$

And

$$X \cap Y = \{(t, \min\{\mu_X(t), \mu_Y(t)\}, \max\{\nu_X(t), \nu_Y(t)\}) \mid t \in E\},$$

The function

$$S(t) = \mu_X(t) - \nu_X(t), t \in E$$

is called “score function”. “It measures the degree of suitability with respect to a set of criteria represented by vague values”.

III. DECISION ON INTUITIONISTIC FUZZY ENVIRONMENT (IFE)

By the effort of Angelov (1997) studied the decision making problem on (IFE). Angelov (1997)proposed model can be described as, consider any set. Let, P_i , “i = 1, 2, ..., h”, are set of h goals and Q_j , “j = 1, 2, ..., k”, are set of k constraints, each of one is defined as an (IFS) on set E. The intuitionistic fuzzy (IF) decision $C = (P_1 \cap P_2 \cap \dots \cap P_h) \cap (Q_1 \cap Q_2 \cap \dots \cap Q_k)$ is Intuitionistic fuzzy set is known as $C = \{(t, \mu_C(t), \nu_C(t)) \mid t \in E\}$, where $\mu_C(t) = \min_{i,j} \{\mu_{P_i}(t), \mu_{Q_j}(t)\}$ and $\nu_C(t) = \max_{i,j} \{\nu_{P_i}(t), \nu_{Q_j}(t)\}$.

Let $S(t) = \mu_C(t) - v_C(t)$, $t \in E$, is “score function” of (IFS) C. T So, $\bar{t} \in E$ is called optimal decision in Intuitionistic fuzzy situation if $S(\bar{t}) \geq S(t)$, for every $t \in E$, that is $S(\bar{t}) = \max_{t \in E} S(t)$. Let ρ, σ are minimal acceptance degree and maximal rejection degree respectively. Angelov (1997) transformed the Intuitionistic fuzzy decision problem into following crisp optimization problem.

Max $\rho - \sigma$

Subject to

$$\mu_{P_i}(t) \geq \rho \quad i = 1, 2, \dots, h$$

$$v_{P_i}(t) \leq \sigma \quad i = 1, 2, \dots, h$$

$$\mu_{Q_j}(t) \geq \rho \quad j = 1, 2, \dots, k$$

$$v_{Q_j}(t) \leq \sigma \quad j = 1, 2, \dots, k$$

$$\rho \geq \sigma \geq 0, \rho + \sigma \leq 1, t \in E$$

3.1 Meaning of $(\alpha \gtrsim \beta)$ in fuzzy

The fuzzy statement means $\alpha \gtrsim \beta$ understand as “ α is essentially greater than or equal to β ” in fuzzy way Zimmermann (1991). To influence the significant choice for membership function argued that $\alpha \gtrsim \beta$ then the inequity is totally satisfied and if $\alpha \leq \beta - s$ where $s > 0$ the inequality is totally disrupted. For $\alpha \in [\beta - s, \beta]$ the membership function is monotonically increasing. Then the membership function is

$$\mu(\alpha) = \begin{cases} 1 & \alpha \geq \beta \\ 1 - \frac{\beta - \alpha}{s} & \beta - s \leq \alpha < \beta \\ 0 & \alpha < \beta - s \end{cases}$$

The inequality understands as “ α is essentially greater than or equal to β with tolerance s ”. Now, $\alpha \gtrsim_s \beta$ is the fuzzy inequality $\alpha \gtrsim \beta$ to the tolerance level d .

Meaning of $(\alpha \gtrsim \beta)$ in Intuitionistic fuzzy

The meaning of inequality $\alpha \gtrsim \beta$ in intuitionistic fuzzy, which is denoted by $\alpha \gtrsim^{IF} \beta$ and is characterized as

$$\{(t, \mu(t), v(t)) \mid t \in E\}$$

The inequality $\alpha \gtrsim \beta$ has two approach that is the pessimistic approach or optimistic approach.

3.2 Pessimistic approach for membership function

In pessimistic approach the decision creator has a pessimistic caution for approval. If the rejection degree of α is zero, the decision maker never accepted fully. Then this scenario can be represented, we assume two tolerance $s, d, 0 < d < s$, be known priori, and described as

$$\mu(\alpha) = \begin{cases} 1 & \alpha \geq \beta \\ 1 - \frac{\beta - \alpha}{s} & \beta - s \leq \alpha < \beta \\ 0 & \alpha < \beta - s \end{cases}$$

And

$$\nu(\alpha) = \begin{cases} 1 & \alpha \leq \beta - s \\ 1 - \frac{\alpha - \beta + s}{d} & \beta - s < \alpha \leq \beta - s + d \\ 0 & \alpha > \beta - s + d \end{cases}$$

To observe the interval $[\beta - s + d, \beta]$ where the belongingness degree is not zero or the non-belongingness degree function is zero at this time the decision maker rejected the inequality and not accepted completely.

3.3 The optimistic approach for membership function

In this the decision maker take generous opinion on refusal. If the acceptance degree of α is zero, then it is not rejected completely by the decision maker and this scenario can be represented, we assume tolerances $s, d > 0$, and defined as

$$\mu(\alpha) = \begin{cases} 1 & \alpha \geq \beta \\ 1 - \frac{\beta - \alpha}{s} & \beta - s \leq \alpha < \beta \\ 0 & \alpha < \beta - s \end{cases}$$

And

$$\nu(\alpha) = \begin{cases} 1 & \alpha \leq \beta - s - d \\ 1 - \frac{\alpha - \beta + s + d}{s + d} & \beta - s - d < \alpha \leq h \\ 0 & \alpha > h \end{cases}$$

Observe that the interval $[\beta - s - d, \beta - s]$ has belongingness degree is zero but the non-belongingness degree is not equal to zero and at this time the decision maker accepted the inequality and not rejected completely.

IV. INTUITIONISTIC FUZZY LINEAR PROGRAMMING DUALITY (IFLPD)

Bector and Chandra (2002, 2005), Wu (2003) or many researcher suggested many methods to study fuzzy linear programming duality (FLPD). The methods totally based upon the kind of fuzziness existing in the model, that is either the fuzzy goals, or the fuzzy parameters, and both the goals and the parameters are fuzzy. Aggarwal (2012) developed the duality theory for (FLPP) where only the goals are (IF) and relate with the “aspiration level approach” by Zimmermann (1991) and Bector (2004).

Let \mathbb{R}^n denote the Euclidean space of n-dimensional and \mathbb{R}_+^n be its non-negative orthant. Let $u \in \mathbb{R}^n$, $v \in \mathbb{R}^m$, and $L \in \mathbb{R}^{m \times n}$. The model has aspiration level X_0 .

(IFP)

Find $y \in \mathbb{R}^n$

such that

$$u^T y (IF) \gtrsim X_0$$

$$Ly (IF) \lesssim v$$

$$y \geq 0$$

For dual to (IFP)

(IFD)

Find $z \in \mathbb{R}^m$

Such that

$$v^T z (IF) \lesssim Z_0$$

$$L^T z (IF) \gtrsim u$$

$$z \geq 0$$

here for dual objective Z_0 is an aspirational level. The relationship of duality between (IFP) and (IFD) depends upon the definite attitude described the (IF) inequalities. Aggarwal et al. (2012) proposed intuitionistic fuzzy inequalities in (IFP) and (IFD) are for pessimistic approach.

V. DUALITY (PESSIMISTIC APPROACH)

Suppose the Intuitionistic fuzzy primal problem (IFPP). Suppose that, $s_i, t_i, 0 < s_i < t_i$, “ $i = 0, 1, \dots, m$, are tolerances respectively with the acceptance and the rejection of $m + 1$ constraints in (IFP). Let us assume that ρ, σ are minimal degree of acceptance and maximal degree of rejection respectively of the constraints $m+1$ of the primal problem (IFP)”. Angelov

(1997) defines the crisp optimization problem is equal to the intuitionistic fuzzy optimization problem (IFP) under pessimistic situation.

(IFPC)

$$\text{Max } \rho - \sigma$$

Subject to

$$(1 - \rho)s_0 + u^T y - X_0 \geq 0$$

$$(1 - \rho)s_i - L_i y + v_i \geq 0 \quad i = 1, \dots, m$$

$$(1 - \sigma)t_0 - u^T y + (X_0 - s_0) \leq 0$$

$$(1 - \sigma)t_i + L_i y - (v_i + s_i) \leq 0 \quad i = 1, \dots, m$$

$$\rho \geq \sigma \geq 0, \rho + \sigma \leq 1, y \geq 0$$

Observe that for $s_i = t_i$, “ $i = 0, 1, \dots, m$ ”, and $\sigma = 1 - \rho$, difficulties (IFP) and (IFPC) reduces to the “standard primal fuzzy linear problem and it is equivalent crisp problem studied by Bector and Chandra” (2004).

Now let us assume intuitionistic fuzzy dual problem (IFD). Let “ $a_j, b_j, 0 < b_j < a_j, j = 0, 1, \dots, n$ are tolerances respectively with the acceptance and rejection of the $n+1$ constraints in IFD. Let ϕ and χ be the minimal degree of acceptance and maximal degree of rejection of $n + 1$ constraints in IFD”.

(IFDC)

$$\text{Max } \phi - \chi$$

Subject to

$$(1 - \phi)a_0 - v^T z + Z_0 \geq 0$$

$$(1 - \phi)a_j + L_j^T z - v_j \geq 0 \quad j = 1, \dots, n$$

$$(1 - \chi)b_0 + v^T z - (Z_0 + a_0) \leq 0$$

$$(1 - \chi)b_j - L_j^T z + (v_j - a_j) \leq 0 \quad j = 1, \dots, n$$

$$\phi \geq \chi \geq 0, \phi + \chi \leq 1, z \geq 0$$

Now observe that for $a_j = b_j$, “ $j = 0, 1, \dots, n$ ”, and $\chi = 1 - \phi$, difficulties (IFD) and (IFPC) reduces to the “standard dual fuzzy problem and it is equivalent crisp problem studied by Bector and Chandra” (2004).

Aggarwal et al. (2012) proposed “duality theorems for IFPC and IFDC”.

Theorem: Let (y, ρ, σ) and (z, ϕ, χ) be possible results for (IFPC) and (IFDC).

Then,

$$(\rho-1)s^Tz + (\phi-1)a^Ty \leq b^Tz - u^Ty,$$

$$(\sigma-1)t^Tz + (\chi-1)b^Ty \geq (u-a)^Ty - (v+s)^Tz,$$

where $s = (s_1, \dots, s_m)^T$, $t = (t_1, \dots, t_m)^T$, $a = (a_1, \dots, a_n)^T$, $b = (b_1, \dots, b_n)^T$.

Remark 1 The (IFPC) and (IFDC) for first and the third constraints are,

$$(\rho-1)s_0 + (\phi-1)a_0 \leq (X_0 - W_0) + (u^Ty - v^Tz), \tag{A}$$

$$(\sigma-1)t_0 + (\chi-1)b_0 \geq (v^Tz - u^Ty) + (X_0 - Z_0) - (s_0 + a_0). \tag{B}$$

Then, “the inequality in (A) relates the comparative dissimilarity of aspiration levels X_0 of u^Ty and Z_0 of v^Tz in terms of their degree of membership and tolerance levels”. While (B) relates the comparative dissimilarity between the minimum aspiration level $(X_0 - s_0)$ of u^Ty and maximum aspiration level $(Z_0 + a_0)$ of v^Tz in terms of their non-membership degree and tolerance levels as definite by the decision maker”.

Remark 2 “It is observed that the crisp problems (IFPC) and (IFDC) do not create a primal-dual pair in the conventional sense of duality in linear programming but are dual in intuitionistic fuzzy sense”. Thus if $(\bar{y}, \bar{\rho}, \bar{\sigma})$ is optimum for (IFPC) or

$(\bar{z}, \bar{\phi}, \bar{\chi})$ is optimum for (IFDC), and never except $\bar{\rho} - \bar{\sigma} = \bar{\phi} - \bar{\chi}$.

VI. TPZSMG WITH IFG AND (TPZSMGIFG):

The duality theory developed previously issued for learning and designing of TPZSMGIF aspiration levels and tolerances for two players.

Suppose $A \in R^{m \times n}$ is $m \times n$ matrix of real number and $e = (1, \dots, 1)^T$ is a identity vector having value one and measurement is specified in the definite environment. A TPZSMG G , having the triplets $G = (S^m, S^n, A)$,

Where $S^m = \{y \in R_+^m \mid e^Ty = 1\}$ and $S^n = \{x \in R_+^n \mid e^Tx = 1\}$ be the approach of player I and player II respectively and A is a payoff matrix. So, $y \in S^m$ and $x \in S^n$, the scalar $y^T Ax$ is payoff of player I and $-y^T Ax$ is payoff of player II if the game is zero sum. Aggarwal et

al. (2012) introduce the IFMG, where the aspiration levels of player I and player II is P_0 and Q_0 respectively.

Thus

$$\text{IFG} = (S^m, S^n, A, P_0, (IF) \gtrsim, Q_0, (IF) \lesssim)$$

Here all fuzzy inequality is taken as intuitionistic fuzzy sense that is pessimistic and optimistic. Aggarwal et al. introduce a new solution on IFG.

Definition: An element $(\bar{y}, \bar{x}) \in S^m \times S^n$ is called a solution of the IFG if

$$\bar{y}^T A x (IF) \gtrsim P_0, \quad \forall x \in S^n$$

$$y^T A \bar{x} (IF) \lesssim Q_0, \quad \forall x \in S^m$$

Let s_0 and t_0 respectively be the “tolerances pre-specified by player I for accepting and rejecting the aspirational level P_0 and a_0 and b_0 respectively be the tolerances pre-specified by player II for accepting and rejecting the aspiration level Q_0 ”. Let us assume that ρ, σ are “minimal degree of acceptance and maximal degree of rejection respectively of the constraints of (IFG1) and ϕ and χ be the minimal degree of acceptance and maximal degree of rejection respectively of the constraints of (IFG2)”. Aggerwal et al. (2012) introduce that the two IFLPP are equal to the following two crisp optimization problems respectively.

(CFP1)

$$\text{Max } \rho - \sigma$$

Subject to

$$(1 - \rho)s_0 + A_j^T y - P_0 \geq 0 \quad j = 1, 2, 3, \dots, n$$

$$(1 - \sigma)t_0 - A_j^T y + (P_0 - s_0) \leq 0 \quad j = 1, 2, 3, \dots, n \quad y \geq 0, \sum_{i=1}^m y_i = 1$$

$$\rho \geq \sigma \geq 0, \rho + \sigma \leq 1$$

(CPF2)

$$\text{Max } \phi - \chi$$

Subject to

$$(1 - \phi)a_0 - A_i z + Q_0 \geq 0 \quad i = 1, 2, 3, \dots, m$$

$$(1 - \chi)b_0 + A_i z - (Q_0 + a_0) \leq 0 \quad i = 1, 2, 3, \dots, m$$

$$z \geq 0, \sum_{j=1}^n z_j = 1$$

$$\phi \geq \chi \geq 0, \phi + \chi \leq 1$$

Where $A_{.j}$ is the j^{th} column and $A_{i.}$ is the i^{th} row of A.

By “solving the intuitionistic fuzzy matrix game IFG is equivalent to solving two crisp optimization problems CFP1 and CFP2 for player I and player II respectively and if $(\bar{y}, \bar{\rho}, \bar{\sigma})$ is optimal solution for CFP1 then we take optimal strategy, optimal degree of acceptance and optimal degree of rejection for player I aspiration level P_0 respectively and $(\bar{z}, \bar{\phi}, \bar{\chi})$ is optimal solution for CFP2 then we take optimal strategy, optimal degree of acceptance and optimal degree of rejection for player II aspiration level Q_0 respectively”. Aggarwal et al. (2012) summarized all in the form of theorem.

Theorem: The IFG described as

$$(S^m, S^n, A, P_0, (IF) \gtrsim, s_0, t_0, Q_0, (IF) \lesssim, a_0, b_0)$$

Is “equivalent to two crisp linear programming problems (CFP1) and (CFP2) which constitute a primal-dual pair in the sense of duality for linear programming in intuitionistic fuzzy environment”.

Based on fuzzy *linear programming duality problems* by Bector et al. (2004) the TPZSMG with “fuzzy goals” and pair of “primal-dual fuzzy linear programming problems” and solve some example and introduce that there would not be any “strong duality between pair of (FLP) and optimal solution for FLP and FLD might not be equal. These fuzzy games theoretical result are not same. Now based on LP with IFSMG with intuitionistic fuzzy goals by Aggarwal et al. (2012) solve the effects of Bector et al. (2004) with Intuitionistic fuzzy game. The optimal solutions of the above results of Bector et al. (2004) solving with Intuitionistic fuzzy game in same fuzzy situation, there is no increase in the acceptance degree.

VII. CONCLUSION

We conclude that the primal-dual (FLP) problems and Intuitionistic fuzzy inequalities depending upon the pessimistic and optimistic approach of the decision maker. Aggarwal et al. (2012) solving (TPZSMG) with intuitionistic fuzzy goals are equivalent to solving two (IFLPP) which are dual to each other in intuitionistic fuzzy sense. The matrix games are assumed to have pessimistic viewpoints, the result can be recognized for optimistic case as well.

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