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An Enhanced Algorithm for Solving Fuzzy Transportation Problems Using Modified SVAM

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Abstract:

The fuzzy transportation problem constitutes a specialized class of fuzzy linear programming problems (LPPs), which is designed to minimize total fuzzy transportation cost, and satisfy the constraints of both supply and demand. The fuzzy optimization models incorporate fuzzy constraints and objectives, enabling decision-makers to make more robust and flexible transportation plans. As a fundamental optimization model in logistics and operations research, it seeks an optimal distribution strategy that ensures all supply sources as well as demands meet requirements at the lowest possible cost through finding an initial basic feasible solution (IBFS). This paper puts forward a new transportation method called (MSVAM). The unique contribution of the new proposed method lies in minimizing the total transportation cost compared to other existing methods, while reducing the time and effort of the computational process. To validate the effectiveness and applicability of the proposed method, a number of examples (one trapezoidal and the other hexagonal) are solved using MSVAM, and compared with other contemporary methods.

Keywords: Transportation Problem, FTP, IBFS, VAM, SVAM, Robust Ranking Method, MSVAM.

I.INTRODUCTION

The transportation problem (TP) is a fundamental optimization challenge in operations research and supply chain management. It focuses on minimizing the cost of distributing goods from multiple suppliers (sources) to various consumers (destinations) while satisfying supply and demand constraints. Traditional TP models rely on crisp, deterministic parameters—fixed transportation costs, exact supply capacities, and precise demand values. However, real-world logistics often involve uncertainty, imprecision, and vagueness due to dynamic market conditions, fluctuating costs, or incomplete data. The fuzzy set theory, introduced by Zadeh (1965), provides a powerful mathematical framework for handling incomplete or ambiguous information using membership functions. In a standard transportation problem, the goal is to find an optimal solution for an assignment of goods from suppliers to consumers, minimizing transportation costs while satisfying supply and demand constraints. The fuzzy transportation problem (FTP) is an extension of the classical TP which introduces uncertainty into the problem by representing elements such as supply, demand, and transportation costs as fuzzy numbers or fuzzy sets. Membership functions are at the heart of fuzzy set theory—they define how "fuzzy" a set really is. A membership function defines how an element belongs to a fuzzy set with a certain degree of membership, unlike classical (crisp) sets where an element either fully belongs (1) or does not belong (0). In other words, it can be thought of as the rulebook that tells us how much an element belongs to a fuzzy set, with values ranging from 0 (completely out) to 1 (fully in). The shape of a membership function matters because it directly impacts how a fuzzy system interprets and processes uncertainty. Some of the most common shapes are triangular, trapezoidal, Gaussian, etc. We call the range of possible input values the "universe of discourse" - essentially all the numbers we might need to consider. Visually, you can picture this as a graph where the curve shows how strongly each input value belongs to our fuzzy set.

Key Components of Fuzzy Transportation Problem:

i. **Fuzzy Supply and Demand:** In a fuzzy transportation problem, the demand and supply at each destination and source are represented as fuzzy sets. This means that instead of fixed values, we have degrees of fulfilment or demand satisfaction, reflecting the uncertainty in these quantities.

ii. **Fuzzy Transportation Costs:** Similarly, transportation costs between suppliers and consumers are represented as fuzzy sets, considering the uncertainty in shipping rates, fuel costs, and other variables affecting transportation expenses.

iii. **Fuzzy Objective Function:** The objective function in a fuzzy transportation problem aims to minimize/reduce the total cost while taking into account the fuzzy nature of both supply, demand, and transportation costs.

In this paper, we have developed an algorithm to solve FTP to obtain better IBFSs in less computational time by modifying SVAM model which is actually inspired by VAM and purposed to solve transportation problems in a shorter time than VAM. Our paper is structured as follows: Section 2 presents a short survey of the literature review related to fuzzy transportation problem. In Section 3, we have provided some important definitions concerned with FTP. In Section 4, we have illustrated the mathematical formulation. In Section 5, we have put forward the proposed method for finding IBFS for a fuzzy transportation problem. Section 6 presents two examples in support of the applicability and practicality of our proposed method. Section 7 summarizes the results of our article, whereas section 8 concludes the paper.

Literature Review

The fuzzy transportation problem (FTP) extends the classical transportation problem by incorporating uncertainty in supply, demand, and transportation costs using fuzzy set theory (Zadeh, 1965). Over the past few decades, researchers have developed various models and algorithms to solve FTPs. This review categorizes key contributions into theoretical foundations, solution methods, and applications.

It was L. A, Zadeh (1965) who introduced fuzzy set theory to handle imprecise data in mathematical models. He defined membership functions to quantify uncertainty (e.g., "approximately 100 units"), and enabled modeling of fuzzy supply, demand, and cost in transportation problems. Bellman & Zadeh (1970) proposed fuzzy optimization for decision-making under uncertainty. They introduced fuzzy constraints and objectives in mathematical programming and formulated FTP as a fuzzy linear programming (FLP) problem. Dubois and Prade (1978) presented a foundational study on algebraic operations involving fuzzy numbers, which are fuzzy subsets of the real line characterized by gradual membership and uncertainty. They extended the classical arithmetic operations—addition, subtraction, multiplication, division, and max/min functions—to fuzzy numbers using a formal fuzzification principle and concluded that this operational framework enables the extension of traditional algorithms (e.g., linear programming, PERT) to contexts involving vague or imprecise data, without significant additional computational burden. Chanas et al. (1984) were the first scholars to introduce the first fuzzy transportation model and to apply fuzzy logic to transportation problems. They proved that a feasible solution exists only if total fuzzy supply = total fuzzy demand and used Interval-valued fuzzy numbers for costs and constraints. Liu &

Kao (2004) presented a methodological framework for determining the objective function value in fuzzy transportation problems where the representation of both cost parameters and supply-demand variables is fuzzy numbers. The proposed approach utilizes Zadeh's extension principle as its foundational theoretical basis. Pandian & Natarajan (2010) introduced a novel approach called the fuzzy zero point method to determine an optimal solution of FTP. The model considers scenarios where transportation costs, supply, and demand are represented as trapezoidal fuzzy numbers. The proposed method guarantees that the resulting solution is also expressed as a trapezoidal fuzzy number, maintaining consistency with the input parameters. Kaur & Kumar (2012) introduced an innovative computational approach designed to solve a specific category of fuzzy transportation problems. The methodology addresses scenarios where decision-makers face uncertainty exclusively in transportation cost parameters, while supply and demand quantities remain precisely known. The algorithm employs generalized trapezoidal fuzzy numbers to model the imprecise transportation costs, providing a robust framework for handling such partial uncertainty conditions. Ebrahimnejad, A. (2016) introduced a novel approach to address fuzzy transportation problems where all parameters (costs, supply, and demand) are modeled as non-negative LR flat fuzzy numbers. The methodology transforms the original problem into four deterministic transportation subproblems, each solvable through conventional simplex-based transportation algorithms. Elumalai (2017) addressed the balanced fuzzy transportation problem using hexagonal fuzzy numbers. Building on earlier work by Pandian, the author proposed an improved Vogel's Approximation Method combined with a Robust Ranking technique to identify the fuzzy optimal solution. The method incorporates the Zero Suffix Method and defines fuzzy membership for the objective function, with both supply and demand expressed as hexagonal fuzzy numbers. Divya Sharma et al (2024) proposed two distinct defuzzification techniques, both grounded in the centroid-based approach, to convert the fuzzy numbers into crisp values suitable for classical optimization techniques. Then, they used the classical Vogel's Approximation Method (VAM) and the Modified Distribution Method (MODI) to derive the initial basic feasible solution and the optimal solution. Aroniadi & Beligiannis (2024) investigated the application of Trigonometric Acceleration Coefficients-Particle Swarm Optimization (TrigAC-PSO) to the Fuzzy Transportation Problem (FTP), which extends the classical Transportation Problem by incorporating uncertainty through fuzzy data. TrigAC-PSO, a modified version of the standard particle swarm optimization algorithm, is evaluated for its effectiveness in solving FTP instances using both classical and generalized fuzzy numbers. Satakshi and Henry (2024) introduced a new method called (SVAM), which is derived from VAM and designed to reduce computation time. Implemented in SciLab (V6.1.1), SVAM is tested across multiple examples and shown to outperform both VAM and LCM in terms of computational efficiency while maintaining solution accuracy.

Although significant advancements have been made in modeling and solving fuzzy transportation problems (FTPs), the critical review of the existing methods discussed above reveals that most efforts focus either on modeling fuzziness using various fuzzy number types (trapezoidal, LR-type, pentagonal, hexagonal) or on proposing defuzzification and solution algorithms such as the fuzzy zero point method (Pandian & Natarajan, 2010), Zadeh's extension principle (Liu & Kao, 2004), and centroid-based transformations (Divya Sharma et al., 2024). While these models enhance the representation of uncertainty, less attention has been given to improving the efficiency of the methods used to obtain an Initial Basic Feasible Solution (IBFS), especially under fuzzy environments.

Traditional approaches such as the Vogel's Approximation Method (VAM) and Modified Distribution Method (MODI) have been widely used, but they often involve extensive computational steps when adapted to fuzzified problems, particularly with higher-order fuzzy numbers like hexagonal and pentagonal types. Despite their widespread use, these classical algorithms do not always yield the least transportation cost and tend to be computationally intensive when handling complex fuzzy data structures.

This study addresses this research gap by proposing a new method called (MSVAM), a method tailored specifically for fuzzy transportation problems. The innovation of the MSVAM lies in its dual objective: not only does it provide a more cost-effective IBFS, but it also significantly reduces computational time and complexity. Unlike existing approaches that prioritize fuzzification or optimality alone, the proposed method emphasizes a balanced improvement in both computational efficiency and cost minimization. The effectiveness of the method is validated through two numerical examples in section 6—one involving trapezoidal fuzzy numbers and another involving hexagonal fuzzy numbers—and benchmarked against VAM and SVAM, confirming the MSVAM's superiority in generating high-quality initial solutions with less computational effort.

Preliminaries

In this section, we introduce the basic concepts used throughout this article.

Fuzzy Set

Zadeh (1965). Let X be a nonempty set of the universe, and x be any particular element of X. The fuzzy set A defined on X can be expressed as a collection of ordered pairs:

$$A= \{(x, \mu A\ (x))\}, x \in X$$

Where μA: X → [0 1] is called the membership function.

Interval Number:

(Moore, R. E., & Yang, C. T. (1959). Let R be the set of real numbers. Then closed interval [a, b] is said to be an interval number, where a, b ∈ R, a ≤ b.

Fuzzy Number

Kaufmann and Gupta (1988). A fuzzy number is defined as a generalization of a conventional real number that represents not a single precise value but a connected set of possible values. Each value in this set is associated with a weight ranging from 0 to 1, known as the membership degree. This weight is defined by the membership function $\mu_{\tilde{A}(x)}$ which satisfies the following conditions:

1. The fuzzy set A defined on the universe of discourse X must be normal, meaning that its height h(A)=1h(A) = 1h(A)=1.
2. The fuzzy set A on X must be convex, ensuring that all intermediate values between any two points in the set have membership degrees at least as high as the minimum of the two.
3. The membership function of A must be piecewise continuous.

Triangular Fuzzy Number

Kaufmann, A., & Gupta, M. M. (1985). A fuzzy number $\widetilde{A}$ = (a, b, c), where a, b, and c are real numbers such that $a \leq b \leq c$ is defined as triangular fuzzy number when its membership function is:

$$\mu_{\tilde{A}(x)} = \begin{cases} \frac{x-a}{b-a}, \ if \ a \leq x \leq b \\ \frac{c-x}{c-b}, if \ b \leq x \leq c \\ 0 \ otherwise \end{cases}$$

Trapezoidal Fuzzy Number

Kaur and Kumar (2012), & Zimmermann, H.-J. (1991). A fuzzy number $\tilde{A} = (a, b, c, d)$ where a, b, c and d are real numbers and a$\leq b \leq c \leq d$ is called a trapezoidal fuzzy number when the membership function is:

$$\mu_{\tilde{A}(x)} = \begin{cases} \frac{x-a}{b-a}, a \leq x < b \\ 1, b \leq x \leq c \\ \frac{x-d}{c-d}, c < x \leq d \\ 0, \quad otherwise \end{cases}$$

Ranking Function

A ranking function R: F(R)→R, where F(R) denotes the set of fuzzy numbers which are defined on the real numbers set, assigns a real number to each fuzzy number so that its natural order is preserved. **Chen, S.-J., & Hwang, C.-L. (1992).**

Ranking Function for Trapezoidal Fuzzy Numbers

Kaufmann, A., & Gupta, M. M. (1988). A ranking function F(R) which maps each fuzzy number into the real line. F(μ) represents the set of all trapezoidal fuzzy number. When R is a ranking function and $\tilde{a}= (a, b, c, d) \in$ F(μ), then $R\ (\tilde{a})=(a + b + c + d)\ /4$.

For two trapezoidal fuzzy number

$\tilde{a}= (a_1, a_2, a_3, a_4)$ and $\tilde{b}= (b_1, b_2, b_3, b_4)$ in F(μ) then,

- $\tilde{a} \leq \tilde{b} \Leftrightarrow R\ (\tilde{a}) \leq R(\tilde{b})$
- $\tilde{a} \geq \tilde{b} \Leftrightarrow R\ (\tilde{a}) \geq R(\tilde{b})$
- $\tilde{a} = \tilde{b} \Leftrightarrow R\ (\tilde{a}) = R(\tilde{b})$

Hexagonal fuzzy number

Wang & Elhag (2006). A hexagonal fuzzy number $\tilde{A}$ is denoted by 6-tuples $\tilde{A}$ = (a, b, c, d, e, f,) where a, b, c, d, e and f are real numbers and $a \leq b \leq c \leq d \leq e \leq f$. The membership function is:

$$\mu_{\tilde{A}(x)} = \begin{cases} \frac{1}{2}\left(\frac{x-a}{b-a}\right) \ if\ a \leq x \leq b \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-b}{c-b}\right) \ if\ b \leq x \leq c \\ 1\ if\ c \leq x \leq d \\ 1-\frac{1}{2}\left(\frac{x-d}{e-d}\right) \ if d \leq x \leq e \\ \frac{1}{2}\left(\frac{f-x}{f-e}\right) \ if e \leq x \leq f \\ 0\ otherwise \end{cases}$$

Robust Ranking Technique:

Yager, R. R. (1981). Let $\tilde{a}$ be a convex fuzzy number. The Robust ranking index is:

$$R\ (\tilde{a})= \int_0^1 0.5(a_\alpha^L a_\alpha^U)d\alpha$$

Where$(a_\alpha^L a_\alpha^U)$ is the α level cut of the fuzzy number $\tilde{a}$ and

$$(a_\alpha^L a_\alpha^U) = \{((b\text{-}a) + a), (d\text{-}(d\text{-}c)\ \alpha)\}$$

The ranking of the objective values in the proposed method is done with this ranking technique. The Robust ranking index R ($\tilde{a}$) gives the representative value of fuzzy number $\tilde{a}$.

Mathematical Formulation of FTP

The mathematical model of the FTP is given in Table 1:

Suppose that there are p numbers of sources and q destination.

Let $\tilde{s}_i$ represent the fuzzy numbers of sources $i\ (i = 1, 2, 3, \dots, p)$

and let $\tilde{d}_j$ represent the fuzzy numbers of destinations $j\ (j = 1, 2, 3, \dots, q)$.

Mathematical model of fuzzy transportation problem is given below:

$$\text{Minimize } \tilde{Z} = \sum_{i=1}^{p} \sum_{j=1}^{q} \tilde{c}_{ij}\, \tilde{z}_{ij} \quad (1)$$

subject to

$$\sum_{j=1}^{q} \tilde{z}_{ij} = \tilde{s}_i,\ i = 1, 2, \dots, p \quad (2)$$

$$\sum_{i=1}^{p} \tilde{z}_{ij} = \tilde{d}_j,\ j = 1, 2, \dots, q \quad (3)$$

$$\tilde{z}_{ij} \geq 0 \quad \forall i, j. \quad (4)$$

Table 1. The Fuzzy Transportation Table

	1	2	…	q	$\tilde{s}_i$
1	$\tilde{c}_{11}$	$\tilde{c}_{12}$	…	$\tilde{c}_{1q}$	$\tilde{s}_1$
2	$\tilde{c}_{21}$	$\tilde{c}_{22}$	…	$\tilde{c}_{2q}$	$\tilde{s}_2$
⋮	⋮	⋮	⋮	⋮	⋮
p	$\tilde{c}_{P1}$	$\tilde{c}_{P2}$	…	$\tilde{c}_{Pq}$	$\tilde{s}_p$
$\tilde{d}_j$	$\tilde{d}_1$	$\tilde{d}_2$	…	$\tilde{d}_q$	$\sum_{i=1}^{p} \tilde{s}_i = \sum_{j=1}^{q} \tilde{d}_j$

Where:

$\tilde{\mathbf{c}}_{\mathbf{ij}}$ is the per unit fuzzy transportation cost for the transport cost of goods from i^{th} source to j^{th} destination.

$\tilde{\mathbf{z}}_{\mathbf{ij}}$ is the quantity transportation from i^{th} source to j^{th} destination.

$\tilde{\mathbf{s}}_{\mathbf{i}}$ is the supply at source **i** and $\tilde{\mathbf{d}}_{\mathbf{j}}$ is the demand at destination **j**.

$\sum_{i=1}^{p} \sum_{j=1}^{q} \tilde{c}_{ij}\, \tilde{z}_{ij}$ is the total (fuzzy) transportation cost.

If $\sum_{i=1}^{p} \tilde{s}_i = \sum_{j=1}^{q} \tilde{d}_j$, then FTP is said to be balanced. That is, the total supply and total demand are the same.
If $\sum_{i=1}^{p} \tilde{s}_i \neq \sum_{j=1}^{q} \tilde{d}_j$, then FTP is said to be unbalanced. That is, there is a discrepancy between total supply and total demand.

Proposed Method for Solving Fuzzy Transportation Problem

VAM, among many other existing methods, is a widely used method for finding the IBFS of a TP. The method is popular because it often finds a solution that is optimal and/or close to the optimal solution. However, the fact is that it involves extensive calculations and computations, a burden that decision-makers have to go through. Similarly, SVAM (Satakshi and Henry (2024) is an iterative method that attempts to address the issue of extensive computational processes in finding IBFS. Our present method is developed to reduce the computational time and effort to the greatest possible extent, and, at the same time, find/obtain better IBFSs. In this section, we elucidate our proposed MSVAM methodology for determining an IBFS of FTP in less computational processes, which helps avoid the burden of extensive computational processes present in existing methods.
The steps of the proposed modified SVAM method are as follows:

- **Step-1:** Consider all the cells of cost, demand, and supply of a fuzzy transportation problem as fuzzy and convert them into crisp values, using Robust Ranking Method indicated above.
- **Step-2:** Ensure that the given FTP is balanced. If it is not balanced, balance it.
- **Step-3:** Calculate the row penalty for each row by multiplying the supply with the difference of highest and lowest cost. Similarly, for each column, determine a column penalty by multiplying the demand with the difference of highest and lowest cost.
- **Step-4:** Choose row or column having the highest value of penalty.

Note: A highest penalty can be considered more than once, if a demand/supply is not fully satisfied in that particular highest penalty row/column.
Assign the maximum possible to the factor (variables) in the chosen row/column having the lowest unit cost. Demand and supply are adjusted accordingly. Once a row or column is fully satisfied, eliminate it from further consideration. If both a row and a column are satisfied at the same time, cross out only one—either the row or column—and assign a zero value to the supply or demand of the other.

1. Stop, when precisely one row or column with no supply or demand is left uncrossed out.

2. Determine the basic variables in the column (row) using the LCM when one column(row) having positive supply (demand) rests uncrossed out. Stop.
3. When there is zero supply and demand in every uncrossed-out row and column, identify the zero basic variables using the LCM. Stop.

Numerical examples:

In order to illustrate the applicability of our suggested method, we have taken two examples from the literature and solved them using MSVAM, SVAM and VAM.

Example I Solaiappan, S., & Jeyaraman, K. (2014).

Table 2:

	D_1	D_2	D_3	D_4	**Fuzzy Supply**
S_1	[-2,0,2,8]	[-2,0,2,8]	[-2,0,2,8]	[-1,0,1,4]	[0,2,4,6]
S_2	[4,8,12,16]	[4,7,9,12]	[2,4,6,8]	[1,3,5,7]	[2,4,9,13]
S_3	[2,4,9,13]	[0,6,8,10]	[0,6,8,10]	[4,7,9,12]	[2,4,6,8]
Fuzzy Demand	[1,3,5,7]	[0,2,4,6]	[1,3,5,7]	[1,3,5,7]	[4,10,19,27]

1. **Solution by VAM.**

Table 3: IBFS using VAM (Example I)

P_1	P_2	P_3	P_4
1	---	---	---
1	1	3	---
0	0	0	0

	D_1	D_2	D_3	D_4	Supply
S_1	2(3)	2	2	1	3
S_2	10	8	5(3)	4(4)	7
S_3	7(1)	6(3)	6(1)	8	5
Demand	4	3	4	4	15

P_1	5	4	4	3
P_2	3	2	1	4
P_3	3	2	1	---

IBFS: X_{11}=3, X_{23}=3, X_{24}=4, X_{31}=1, X_{32}=3, X_{33}=1
Total transportation cost = (2*3) +(5*3) +(4*4) +(7*1) +(6*3) +(6*1) =68

2. **Solution by SVAM:**

Table 4: IBFS using SVAM (Example I)

	D_1	D_2	D_3	D_4	Supply
S_1	2(3)	2	2	1	3
S_2	10	8	5(3)	4(4)	7
S_3	7(1)	6(3)	6(1)	8	5
Demand	4	3	4	4	15

P_1
3
42
10

P_1	32	18	16	28

IBFS: X_{11}=3, X_{23}=3, X_{24}=4, X_{31}=1, X_{32}=3, X_{33}=1
Total transportation cost = (2*3) +(5*3) +(4*4) +(7*1) +(6*3) +(6*1)
=6+15+16+7+18+6=68

3. **Solution by MSVAM**

Table 5: IBFS using MSVAM (Example I)

	D_1	D_2	D_3	D_4	Supply
S_1	2(3)	2	2	1	3
S_2	10	8	5(3)	4(4)	7
S_3	7(1)	6(3)	6(1)	8	5
Demand	4	3	4	4	15

P_1
3
42
10

P_1	32	18	16	28

IBFS: X_{11}=3, X_{23}=3, X_{24}=4, X_{31}=1, X_{32}=3, X_{33}=1
Total transportation cost = (2*3) +(5*3) +(4*4) +(7*1)+(6*3) +(6*1) =68

Example 2:

(Elumalai, P., Prabu, K., & Santhoshkumar, S., 2017) A company has four sources S_1, S_2, S_3 *and* S_4 and four destinations $D_1, D_2, D_3,$ *and* D_4 ;The fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination is C_{ij} where

$$[C_{ij}]_{3*4} = \begin{bmatrix} (5,10,15,20,25,30) & (-5,10,25,40,55,70) & (4,11,18,25,32,39) & (6,12,18,24,30,36) \\ (-4,9,22,35,48,61) & (1,7,13,19,25,31) & (6,9,12,15,18,21) & (6,14,22,30,38,46) \\ (4,8,12,16,20,24) & (5,10,15,20,25,30) & (1,7,13,19,25,31) & (-5,10,25,40,55,70) \end{bmatrix}$$

Table 6:

Source	D_1	D_2	D_3	D_4	Supply
S_1	(5,10,15, 20,25,30)	(−5,10,25, 40,55,70)	(4,11,18, 25,32,39)	(6,12,18 ,24,30,36)	(6,10,14, 18,22,26)
S_2	(−4,9,22, 35,48,61)	(1,7,13, 19,25,31)	(6,9,12, 15,18,21)	(6,14,22, 30,38,46)	(6,10,14, 18,22,26)
S_3	(4,8,12, 16,20,24)	(5,10,15, 20,25,30)	(1,7,13, 19,25,31)	(−5,10,25, 40,55,70)	(14,8,2, -4,-10,-16)
	(4,8,12, 16,20,24)	(-7,7,21, 35,49,63)	(6,9,12, 15,18,21)	(23,4, -15, -34, -53, -72)	(26,28,30, 32,34,36)

Solution:

Convert the given fuzzy problem into a crisp value by using Robust ranking method for all cells as the following: The α-cut of fuzzy number (5,10,15,20,25,30) is (a_α^L, a_α^U) dα= (5α+5, 15-5α) for which

R (5,10,15,20,25,30) = $\int_0^1 (0.5)$ (5α+5, 15-5α) dα

=$\int_0^1 (0.5)(20)$ dα

=10

Similarly,

R (6,10,14,18,22,26) = $\int_0^1 (0.5)(6$ α+6, 14-6 α) dα

= $\int_0^1 (0.5)(20)$ dα

=10

R (4,8,12,16,20,24) = $\int_0^1 (0.5)(4$α+4, 12-4α) dα

= $\int_0^1 (0.5)(16)$ dα

=8

Defuzzify all other fuzzy cells in table 6 in the same way as shown above in order to obtain crisp values as shown in table 7.

Table 7

	D_1	D_2	D_3	D_4	Supply
S_1	10	10	11	12	10
S_2	9	7	9	14	10
S_3	8	10	7	10	8
Demand	8	7	9	4	28

1. Solution by SVAM:

Table 8: IBFS using SVAM (Example 2)

	D_1	D_2	D_3	D_4	Supply
S_1	10(8)	10	11	12(2)	10
S_2	9	7(7)	9(1)	14(2)	10
S_3	8	10	7(8)	10	8
Demand	8	7	9	4	28

P
20
70
24

P	16	21	36	16

IBFS: $X_{11} = 8, X_{14} = 2, X_{22} = 7, X_{23} = 1, X_{24} = 2, X_{33} = 8$

Then the total cost= (10*8) +(12*2) +(7*7) +(9*1) +(14*2) +(7*8) = 80+24+49+9+28+56 = 246

2. Solution by MSVAM

Table 9: IBFS using MSVAM (Example2)

	D_1	D_2	D_3	D_4	Supply
S_1	**10(5)**	**10**	**11(1)**	**12(4)**	**10**
S_2	**9(3)**	**7(7)**	**9**	**14**	**10**
S_3	**8**	**10**	**7(8)**	**10**	**8**
Demand	**8**	**7**	**9**	**4**	**28**

P
20
70
24

P	16	21	36	16

IBFS: $X_{11} = 5, X_{13} = 1, X_{14} = 4, X_{21} = 3, X_{22} = 7, X_{33} = 8$
Total cost= (10*5) +(11*1) +(12*4) +(9*3) +(7*7) +(7*8)
=50+11+48+27+49+56
=241

Results Analysis:

In order to check the efficiency of our proposed method, we have solved two examples from the literature by our proposed algorithm and compared the obtained results with existing approaches like Vogel's Approximation Method (VAM) and SVAM. In the first example, in which we used trapezoidal fuzzy numbers, all three methods—VAM, SVAM, and MSVAM—produced the same initial solution with a total cost of 68, showing comparable performance. However, in the second example involving hexagonal fuzzy numbers, MSVAM outperformed VAM and SVAM by achieving a lower transportation cost (241 compared to 246). The key advantage of MSVAM lies in its computational efficiency—it recalculates penalties for high-priority rows or columns multiple times if supply or demand isn't fully met, leading to better solutions without excessive additional effort. The study also highlights the use of the Robust Ranking Method to convert fuzzy numbers into crisp values, ensuring reliable comparisons. Overall, MSVAM proves to be a flexible and effective method for handling uncertainty in transportation problems, offering improved cost efficiency and adaptability for different fuzzy number types. While the results are promising, further testing on larger or more complex problems could strengthen its validation. This method could be particularly useful in logistics and supply chain management, where decision-makers often face imprecise data.

Fig. 1:

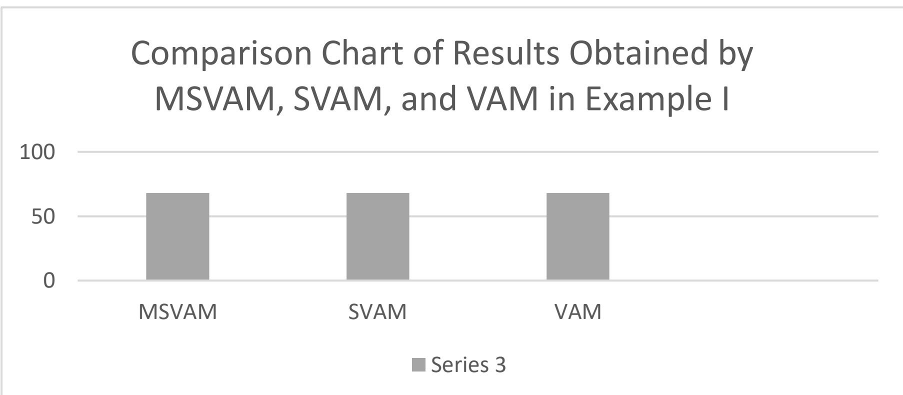
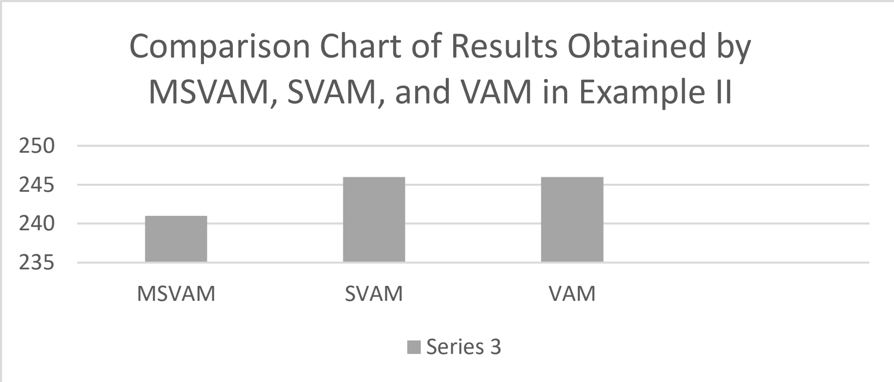


Fig.2:



II. CONCLUSION

The paper presents a new method called MSVAM for finding the IBFS of a fuzzy transportation problem. The proposed method incorporates fuzzy objectives and constraints to handle uncertainty in the transportation problem. Incorporating fuzzy logic in transportation problem-solving can enhance decision support systems, giving way for decision-makers to account for uncertainty and make more informed choices in less computational time. The suggested approach performs better than other existing methods such as VAM and SVAM in terms of the transportation cost and computation time taken for finding IBFS of a transportation problem. In Vogel's Approximation Method (VAM), the penalty for each row or column is calculated by finding the difference between the smallest and the second smallest cost values within that row or column, which takes a long computational time. SVAM calculates the penalty of every row and column only one time, moving from the highest penalty to the lowest penalty. But our proposed method computes the penalty of the highest row and column, bearing in mind that the highest penalty can be considered more than once, if a demand/supply is not fully satisfied in that particular highest penalty row/column. Thus, the proposed method is designed to obtain better IBFS for fuzzy transportation problem than VAM or SVAM in a remarkably less computational time and lesser effort.

III. REFERENCES

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